

Laws of Particle Entrainment from Fluidized Bed Apparatuses

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Abstract: The article presents the results of experimental studies of the process of entrainment of particles of fine fractions from the fluidized bed of polydisperse mixture of granular superphosphate. A review of the main analytical dependencies for determining the amount of entrainment from the fluidized bed is presented. The results of experimental studies of the entrainment process from the fluidized bed of small particles with a size less than 1 mm and large particles with a size more than 1 mm in the interval of gas flow velocities 0.5 - 5.0 m/s are given. The results of calculating the value of fine fraction entrainment from the fluidized bed of superphosphate granules according to the known equations are analysed. The analytical dependence, which is more rational from the practical side, is proposed.

Keywords: granular fertilizers, fine fraction, separation, gas flow, fraction, fine particle concentration, weighted layer, gas flow velocity

1. Introduction

Fluidized bed technology is widespread in chemical, oil refining, fuel and energy, and other industries. In the chemical industry due to active hydrodynamics and high intensity of heat and mass transfer fluidized bed apparatuses are used for drying, cooling, heat treatment, and dedusting of granular materials [1].

In fluidized bed apparatuses, the intensity of phase interaction increases both due to the development of the phase contact surface and due to an increase in the relative velocity of the gas-disperse flow. Inevitable entrainment of fine particles is rightly considered to be one of the main disadvantages of the fluidization technique and should either be minimal in the case of heat and mass transfer processes or regulated by the number and size of particles in the case of dedusting of granular materials. When the volume of dusty waste gas is reduced, the load on dust cleaning equipment is reduced and thus the environmental situation at the production site is improved. Therefore, when calculating and designing fluidized bed apparatuses, an important step is to determine the amount of fine particle entrainment from the fluidized bed of granular material.

2. Problem Statement

The first works on the generalization of experimental data on fine fraction entrainment from the layer were undertaken in the works: Leva [2], Osberg and Charlesworth [3], Kunii and Levenspiel [4]. The authors have experimentally shown that during fluidization an equilibrium between the concentrations of fine particles in the bed and in the entrainment is established for a certain time. The rate of fine entrainment is proportional to its concentration in the bed and is expressed by the dependence of the following form: $\frac{dC}{d\tau} = -k_y C$. (1)

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The entrainment rate of fine particles from the layer is expressed by the equation

$$G_y = k_y G_l C, \quad (2)$$

where: C – concentration of fine particles in the bed, kg/kg; τ – process time, sec; G_y – entrainment rate, kg/s; G_l – mass of particles in the bed, kg; k_y – entrainment rate constant, c^{-1} .

The entrainment rate constant k_y depends on several parameters in the form of a general function [5–7].

$$k_y = f(w, u_s, Fr, Re, d, D, H_l, H_s, \rho, \rho_p), \quad (3)$$

where: w – working speed of fluidization, m/s; u_s – velocity of solid particles in gas flow, m/s; Fr – Froude criterion; Re – Reynolds criterion; d – average diameter of solid particles, m; D – diameter of apparatus, m; H_l , H_s – respectively height of suspended layer and overlayer (separation) space, m; ρ_p , ρ – respectively density of solid particles and gas, kg/m^3 .

In [4], the modified entrainment constant is expressed as:

$$k_{my} = \frac{k_y \cdot G_l}{F_l}, \quad (4)$$

and the influence of the gas flow velocity and the height of the overlayer space on the entrainment (in g/cm^3) of fine particles is expressed by the formula

$$\frac{G_y}{F_l \cdot w} = B \cdot e^{-[(b/w)^2 + a \cdot H_s]}, \quad (5)$$

where: F_l – layer cross-section, cm^2 ; G_y – velocity of fine fraction entrainment, g/s ; w – velocity of gas flow to the free section of the apparatus, cm/s ; H_s – height of the layer space, cm ; B , b , a – experimental constants, which depend on several parameters [4].

In [8, 9] the authors propose to consider the mass fraction of the fine fraction in the layer when determining the entrainment rate constant:

$$k_{my} = \frac{G_y}{c_{sml}}, \quad (6)$$

In [10], the authors experimentally confirm equation (1) and propose a correlation for determining the entrainment rate constant in the form of:

$$k_{my} = 0,36(c_{sml})^{1,09} \cdot \left(\frac{w - u_p}{u_p} \right)^{3,83}, \quad (7)$$

where: c_{sml} – concentration of fine particles in the layer, wt. fraction; u_p – final particle velocity, m/s.

In [11] it is proposed to calculate the intensity of fine particle entrainment from the fluidized bed using the expression:

$$\frac{g_y}{\rho \cdot w} = 6,0 \left(Fr \frac{\rho}{\rho_p} \cdot c_{sml} \right)^{1,37}, \quad (8)$$

where: g_y – flux density of carried away particles, $kg/m^2 \cdot s$.

In formula (8) the Froude criterion is determined by the average diameter of carried away fine particles and the conditions must be met:

$$0,002 < \frac{g_y}{\rho \cdot w} \leq 0,5 \quad (9)$$

$$0,003 < Fr \frac{\rho}{\rho_p} \cdot c_{sml} \leq 2$$

An expression for determining the concentration of fine particles (kg/m^3) in the gas stream of the separation space of the fluidized bed apparatus can be obtained from equation (8) in the form of:

$$y = \frac{G_y}{w} = 6 \frac{w^{2,74} \rho^{2,37} X_l^{1,37}}{g^{1,37} \rho_p^{1,37} d^{1,37}}, \quad (10)$$

where: y – concentration of fine particles in the gas stream, kg/m^3 ; G_y – specific flow rate of fine fraction carried away by the gas stream, $kg/m^2 \cdot s$; w – working speed of fluidization, m/s; X_l – relative content of fine particles in fluidized bed, shares; d – average diameter of solid particles, m; g – acceleration of free fall, m/s^2 ; ρ_p , ρ – density of solid particles and gas, kg/m^3 .

In [12] it is proposed to calculate the intensity of fine particle entrainment from the fluidized bed using a formula of the form:

$$\frac{g_y}{\rho \cdot w} = 0,00845 \cdot Fr^{0,687} \cdot Re^{-0,226} \left(\frac{\rho_p}{\rho} \right)^{0,564}. \quad (11)$$

In formula (11) the Froude criterion is determined by the height of the separation space and the following conditions must be met:

$$150 < (1/Fr) < 1000. \quad (12)$$

The Friedland-Scoblo equation [13] is known to determine the entrainment of the form:

$$\frac{G_y}{G_{gas}} \cdot 10^2 = A \frac{w_p^4 X_l^{0,5} H_l^k}{d^{3,53} H_s^n f} \left(\frac{w_{yc}}{w_0} \right)^4, \quad (13)$$

where: G_{gas} – mass flow rate of gas, kg/s; H_i – height of fluidized bed, mm; H_s – height of separation space, mm; d – weighted average diameter of solid particles, mm; f – live section of the gas distribution grid, shares.; w_{yc} – velocity of the beginning of aluminosilicate catalyst entrainment, m/s; w_y – velocity of the beginning of applied material entrainment, m/s; A, n, k – experimental constants.

The empirical equation of Barsukova [13] was proposed to determine the concentration of fine particles (kg/m³) in the gas flow of the separation space of the fluidized bed apparatus of the following form:

$$y = 0,055 \frac{w^{1,52} \rho^{1,76} D^{0,114}}{\rho_p^{0,76}} \left(\frac{X_i}{d \cdot t} \right)^{0,76}, \quad (14)$$

where: D – apparatus diameter, m; t – spacing between the holes of the gas distribution grid, m.

The analysis of the given calculated dependences shows that they are of a private nature due to several assumptions since the values included in the equations are selected only within the specified limits of change in the conditions of a particular experiment. In addition, the above equations include hard-to-define or ambiguously specified quantities, such as the relative content of fine particles in the fluidized bed, the live section and the pitch of the gas distribution grid. Therefore, the complex type of correlations to determine the entrainment value is the reason for discrepancies in the calculation results.

A simpler form of the equation for determining the entrainment rate is proposed in [14]. It is postulated that the entrainment rate is proportional to the limiting concentration of particles of a given size (Y_{lim}), which can occur in the flow at its full "saturation" with suspended particles, i.e.

$$\frac{dy}{d\tau} = -k \cdot y_{lim}. \quad (15)$$

After the transformations described in [14], we obtain equations that allow us to determine the current value of particle concentration in the gas stream (g/m³) at the exit of the fluidized bed:

$$\lg y = \lg y_{lim} - k_1 / w = a - b / w, \quad (16)$$

where: y – concentration of particles of a given size in the gas stream (entrainment), g/m³; y_{lim} – limiting concentration of particles in the gas stream, g/m³; k, a, b – experimental constants that are a function of

the velocity of particles (u_s) of a given size.

The task of this work is to experimentally reveal the regularities of drift of separate fractions of material from the fluidized bed. It is also necessary to conduct a comparative results analysis of the calculated value of fluidized bed dusty material entrainment.

3. Experimental Materials and Methods

The object of the study is the process of pneumatic separation of a polydisperse mixture of granulated superphosphate. Experimental studies of the separation process were carried out in an apparatus with a cross-section of 50×100 mm and a height of up to 1 m together with the separation space (Fig. 1).

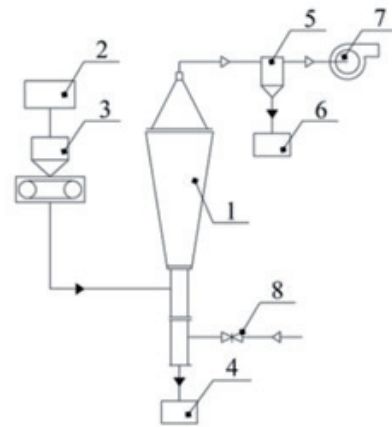


Figure. 1: Schematic diagram of the laboratory pneumatic separation plant.

The laboratory unit (Fig. 1) consisted of a pneumatic classifier 1 equipped with a feed hopper 2 with a belt feeder 3 and a discharge hopper 4. There was a centrifugal cyclone 5 with a discharge hopper 6 for capturing fines. A high-pressure fan 7 provided air pumping through the apparatus and cyclone. The air flow rate was regulated and measured by a calibrated manifold with a control valve 8. The walls of the apparatus from the front side were transparent for visual observation. A perforated gas distribution grid was installed in the working volume of the apparatus, at the level of the feed material inlet.

A polydisperse granular mixture of superphosphate in the form of 0.1-5 mm granules with a shape factor of 0.85 was used in the studies. The granulometric composition of polydisperse

mixture was determined by sieve analysis using a set of wire sieves with aperture sizes (side size of a square hole): 0.5, 1.0, 1.6 and 2.5 mm. The sieves were stacked as vertical block with decreasing hole sizes from top to bottom and placed on a vibrating table with a control unit. The sieve analysis was carried out within 15-20 min, which corresponds to the accuracy of measurements [15]. To evaluate the accuracy of the measurements performed, several samples were analysed according to the methodology [16]. The deviation of the results for each sample did not exceed 1.5-2.0 %. Results of sieve analysis of the initial polydisperse mixture: +2.5 mm - 10 %; -2.5+1.6 mm - 25 %; -1.6+1 mm - 25 %; -1+0.5 mm - 20 %; - 0.5 mm - 20 %. True and bulk densities of superphosphate granules obtained by pelletizing in a drum granulator, according to reference data were taken respectively 2250 kg/m³ and 1100-1200 kg/m³ [17].

The initial mixture of granulated superphosphate with a 3 kg weight was weighed on electronic scales with an accuracy of 0.1 g. The specific material throughput was 6-10 kg/(m²·s). The moisture content of the pellets was less than 1% mass, and the temperature of the pellets corresponded to the ambient temperature. These conditions do not affect the heat transfer process [18]. The air temperature at the inlet to the apparatus was equal to the air temperature in the laboratory room: in summer conditions 22-27 °C; in winter conditions 18-22 °C. The initial material was fed by a feeder into the apparatus and weighed by air flow on a gas distribution grid with a free cross-section of 30%. The weighing mode was a failure. That is, the coarse fraction (particle size more than 1 mm) fell through the grid openings, gravitationally fell down the apparatus, and accumulated in the discharge hopper. The fine fraction (particle size less than 1 mm) was carried away by the air stream and collected in the cyclone. When the apparatus was operating under steady-state conditions (set in 15-20 s after the feeder was switched on), up to 5-6 samples were taken from the discharge hoppers of the apparatus and cyclone. In order to eliminate the sucking of external air during the sampling of material, the discharge hoppers were made double. Spring-loaded valves were installed on the outlets of the upper hoppers. Samples were weighed on electronic scales with an accuracy of 0.1 g. For sieve analysis, the arithmetic mean of the weights

of the selected samples was taken. To evaluate the accuracy of the measurements made, several samples taken in repeated experiments under the same conditions were analysed. The deviation of measurement results for each fraction between the selected samples was 1.5-2.0 %. To eliminate the influence of random factors on the reliability of measurement results, six experiments were carried out each.

The pressure drop of the apparatus was measured with an alcohol U-shaped manometer, one tube of which was connected to the point of the body under the gas distribution grid, and the second tube - at the outlet of the gas flow after the fluidized bed. The measurement error in counting two levels (on each tube) was ±2 mm at an ambient temperature 20±5 °C.

The concentration of fine fraction in the gas flow in the separation zone was determined by the formula:

$$y = \frac{M}{\tau \cdot V_{air}}, \quad (17)$$

where: M – mass of the suspension of the fraction with the size less than 1 mm, carried away to the cyclone during the experiment, g; τ – experiment time, s; V_{air} – air flow rate through the apparatus, m³/s.

4. Results

Table 1 presents the results of experimental studies on the gas flow entrainment of different fractions of material from the fluidized bed. Measurements were carried out for three fractions that were carried away by the gas flow from the fluidized bed. These are fractions with particle sizes less than 0.5 mm (-0.5); less than 1 mm and greater than 0.5 mm (-1.0+0.5); and greater than 1.0 mm (+1.0). To reveal generalized patterns of particle entrainment from the fluidized bed, the experiments were carried out in a wide range of gas flow velocities - from 0.5 to 5.0 m/s.

As the results of experiments show, when increasing the gas flow velocity from 0.5 m/s to 2.0 m/s, the entrainment of dust particles of 0.5 mm fraction monotonically increases on average by 1.5 times, and at the gas flow velocity greater than 2.0 m/s the entrainment growth rate begins to decrease. At a velocity of more than 3.5 m/s, stabilization of the value of dust particle entrainment is observed.

This proves the hypothesis of saturation of the gas flow with suspended particles, that at a certain boundary velocity of the gas flow, there is a limiting concentration of carried away particles.

The entrainment of larger particles of fine particle fraction $-1.0+0.5$ mm starts at gas flow velocity of more than 2.5 m/s. The entrainment growth rate for this fraction is greater than for dust particles. Stabilization of entrainment starts at gas flow velocities more than 5.0 m/s.

The entrainment of coarse fraction $+1.0$ mm starts at gas flow velocity of 3.0 m/s. The growth rate of entrainment is also significant. Stabilization of entrainment begins at gas flow velocities greater than 5.0 m/s.

5. Discussion

Thus, the results of experimental studies prove

that during the processing of granular mineral fertilizers in fluidized bed apparatuses (drying and cooling processes), the separation process, i.e. separation of dust-like particles less than 0.5 mm in size from the polydisperse mixture, is quite effective. In this case, the optimal range of gas flow velocity is within 1.5 - 2.5 m/s. At higher gas flow velocities, over 3.0 m/s, the efficiency of the dedusting process decreases, and energy costs for pumping more gas through the layer increase. At the same time large fractions of more than 1 mm start to drift away. This is unacceptable because the coarse fraction is a finished product, which goes to packaging.

The connection of constants "a" and "b" in equation (16) with the velocities of gas flow and particles' vibration was established experimentally [14] by studying the dependence of distribution of particle concentration value of granulated

Table 1: Experimental results on particle entrainment from fluidized bed

| Material | Fraction, mm | Equivalent particle diameter, mm | Vortex velocity of equivalent particle diameter, m/s | Gas flow velocity, m/s | Concentration of particles in the gas stream separation zone, g/m ³ |
|---------------------------|--------------|----------------------------------|--|------------------------|--|
| Granulated superphosphate | -0.5 | 0.25 | 1.45 | 0.5 | 30 |
| | | | | 0.75 | 60 |
| | | | | 1.0 | 80 |
| | | | | 1.25 | 95 |
| | | | | 1.5 | 120 |
| | | | | 2.0 | 168 |
| | | | | 2.5 | 190 |
| | | | | 3.0 | 218 |
| | | | | 3.5 | 232 |
| | | | | 4.0 | 240 |
| | | | | 4.5 | 248 |
| | | | | 5.0 | 255 |
| | -1.0+0.5 | 0.75 | 4.1 | 2.5 | 5 |
| | | | | 3.0 | 22 |
| | | | | 3.5 | 65 |
| | | | | 4.0 | 130 |
| | | | | 4.5 | 210 |
| | | | | 5.0 | 260 |
| | +1.0 | 2.88 | 9.1 | 3.0 | 5 |
| | | | | 3.5 | 18 |
| | | | | 4.0 | 75 |
| | | | | 4.5 | 160 |
| | | | | 5.0 | 220 |

superphosphate mono-fractions on the height of separation space of fluidized bed apparatus.

The analysis of experimental data has shown [14] that there is a functional relationship between the constants "a" and "b" of equation (16) and the hovering velocity in the form of equations:

$$\begin{aligned} a &= 5,0 \cdot 10^{-\left(\frac{0,75}{u_s}\right)}, \\ b &= 0,15 \cdot u_s^{0,51}, \\ \text{at } u_s &\leq 5,1 \text{ M/c.} \end{aligned} \quad (18)$$

The results of calculations on the entrainment of fine particles of granulated superphosphate (fraction less than 1 mm) from the fluidized bed apparatus (Table 2). Calculations were carried out for industrial conditions, i.e. it was assumed that the cooler of granulated superphosphate has an equivalent diameter of 4 m, live section of gas distribution grid 5% (step 0.12 m), height of fluidized bed 0.3 m - 0.5 m and height of separation space 2 m.

Table 2: Results of calculations of entrainment value

| Flow velocity, m/s | Entrainment, kg/m ³ | | | |
|-----------------------|-----------------------------------|-------------|-------|-------|
| | according to the equations | | | |
| | (10) | (13) | (14) | (16) |
| 1.8 | 0.101 | 0.012-0.014 | 0.207 | 0.239 |
| 2.0 | 0.135 | 0.018-0.023 | 0.243 | 0.246 |
| 2.2 | 0.175 | 0.027-0.032 | 0.281 | 0.251 |
| 2.6 | 0.268 | 0.052-0.063 | 0.362 | 0.261 |

As follows from the data in the table, the results of calculating the value of entrainment by equation (16) satisfactorily coincide with the results of calculations by equations (10) and (14) with a relative error of 3-30 %. Calculation by equation (13) gives an order of magnitude underestimated results, which is explained by some inconsistency of this formula: there is an unreasonably high value of the degree index at the height of the separation space. Equations (13) and (14) include design and mode parameters (live section and grid pitch, apparatus diameter, bed, and separation space height), which can be varied within a wide range, which greatly affects the accuracy of calculations.

Thus, the practical confirmation of the proposed equation (16) allows us to consider that the established experimental dependence correctly enough reflects the influence of physical properties of the gas flow and solid phase on the kinetics of

fine particle entrainment from the fluidized bed. Knowledge of the value of dustiness of the gas flow at the outlet of fluidized bed apparatuses will make it possible to propose ways to reduce the level of gas-dust emissions into the environment.

However, the applicability of equations (16) to (18) is constrained by the particle size range, their physicochemical properties, and the polydispersity of the coarse-grained material mixture. Examples include granular phosphate fertilizers such as superphosphate and ammophos. These equations will not be applicable to powdered products, including various inorganic salts, metal powders, and similar materials.

The investigation of the regularities of small fraction entrainment from a polydisperse mixture of powdered materials will be the focus of future research.

6. Conclusions

1. The equations for determining the value of fine particle amorphous entrainment from the fluidized bed are proposed.

2. It is experimentally shown that at gas flow velocities up to 2 m/s the entrainment of fine particles up to 0.5 mm from the fluidized bed monotonically increases, then the growth rate slows down and stabilizes at a velocity of 3.5 m/s. The growth rate of entrainment of particles larger than 0.5 mm stabilizes at a gas flow velocity of 5 m/s.

3. It is noted that in fluidized bed apparatuses the separation process, i.e. separation of dust-like particles with the size less than 0.5 mm from polydisperse mixture, is carried out quite effectively. In this case, the optimum range of gas flow velocity is within the range of 1.5 - 2.5 m/s.

4. We analysed the results of calculating the value of entrainment by some known equations, which allowed us to designate the most practical equation.

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