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Analysis of the Mixing of a Newtonian Material with a Strongly Viscous Material using CFD Numerical Modelling

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Abstract: This article deals with the mixing of a Newtonian material with a strongly viscous (non-Newtonian) material in a pipe with a static mixer. This problematic has a practical application in the food industry. CFD software ANSYS Fluent was used for numerical modeling. The mathematical model is defined using the basic balance equations, including the equation for species. The viscosity for a non-Newtonian highly viscous material (the working material) is defined using the Herschel-Bulkley model. A 3D computational model of a pipe with a static helical mixer is created in the DesignModeler program, which is part of the ANSYS software. The computational mesh was created in the Fluent Meshing program. The CFD modeling analysis is implemented for two different pipe diameters and different flow rates at the pipe inlet. The evaluation of the results is processed by the ANSYS Fluent program using the distribution of basic quantities and graphs.

Keywords: numerical modelling, strongly viscous substance, static mixer

1. Introduction

Mixing material is an important part of many industries. In the mixing process, the effort is to achieve full homogenization of the working mixture. Tanks and containers are most often used for mixing in the food industry. However, this application is not suitable for all types of production. From an economic and technological point of view, mixing the working material with water in the pipe using mixers can be used as a suitable solution. It is applied if the given working material are transported under pressure in the pipe. Mixing can already occur during flow in an empty pipe. To support mixing, a so-called static mixer is added to the pipe (Figure 1). The advantage of a static mixer is mainly easy maintenance and simple production. It uses external energy to mix the material. A mixer consists of a fixed obstacle, a series of obstacles or a mixing element. It has different production geometries, which can also be combined. The flow of the working material passing through this mixer hits the mixing elements and the flow is divided. There is a distribution of the velocity components in the radial and tangential directions. We use static mixers for all types of flow (laminar, turbulent). The work deals with a similar issue using CFD numerical modeling [1]. Other works discuss the transport of strongly viscous material in similar applications [2], [3], [4].

2. Static mixer model

A helical static mixer was chosen for the analysis of the mixing of the working material with water. The model was created in DesignModeler program. Two pipes of different diameters (TR219x6.3 and TR114x3.6) were created. The pipe TR219 has a

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Figure 1: Helical static mixer.

diameter of 206.4 mm and a length of 3376.5 mm (Figure 2). The pipe TR114 has a diameter of 106.8 mm and a length of 2000 mm (Figure 3). A helical mixer is fixed in each pipe, formed by 6 elements. The thickness of the element was chosen to be 2 mm. The total length of the elements in the pipe TR219 is 2400 mm. The mixer in the pipe TR219 is located at a distance of 800 mm from the inlet of pipe, in the pipe TR114 it is located 500 mm from the inlet of pipe. In the pipe TR114 the total length of the elements is 1200 mm. A tube is placed in the axis of pipe from which a Newtonian material (water) flows. The diameter of tube for the pipe TR219 is 10 mm and length are 206.4 mm. In the pipe TR114 tube has a diameter of 5 mm and a length of 106.8 mm. A strongly viscous material on the other hand enters the pipe through the annulus.

For the subsequent analysis of the mixing process, a geometry with a different position of the water inlet was created, when this inlet from the tube was moved to 50 mm from the mixer (Figure 4). The lengths of the individual pipes were shortened to 3000 mm (TR219) and 1600 mm (TR114).

After the geometry was created, a computational mesh was created in the Fluent Meshing program (Figure 5). The cell size in this mesh was set from 0.005 to 0.01 mm with a growth factor of 1.1. The mash has three boundary layers with a transition ratio of 0.1. The volumec mesh is formed by polyhedral elements. For the pipe TR219 it has 463,067 cells, for the pipe TR114 it has 423,862 cells.

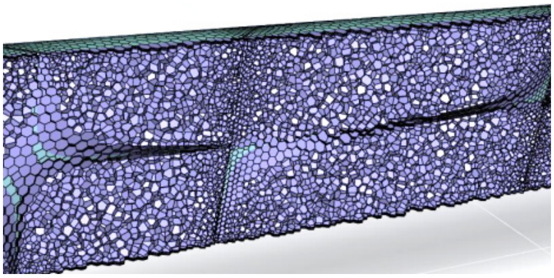

Figure 5: Detail of the helical mixer mesh.

3. Mathematical model for CFD analysis in ANSYS Fluent

The mathematical model of the fluid flow of the mixture (working material, water) is defined by differential equations that express the basic laws [5], [6]. The law of conservation of mass is being represented by the continuity equation. Expression in differential vector form:

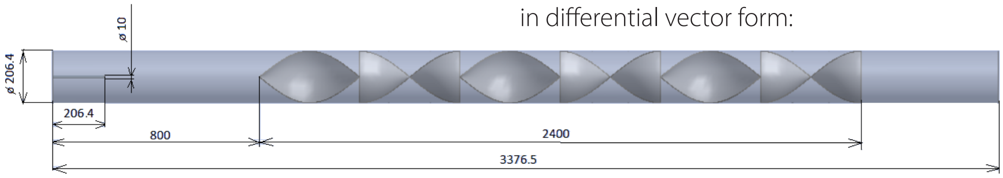

Figure 2: Pipe dimensions TR219x6.3 with helical mixer.

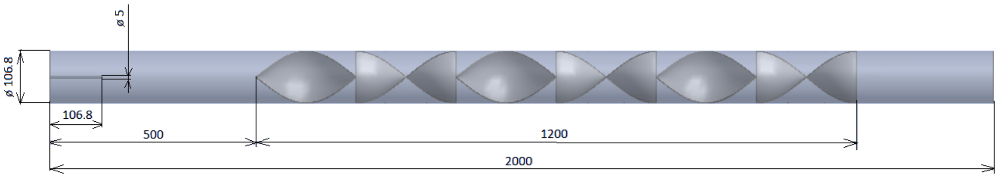

Figure 3: Pipe dimensions TR114x3.6 with helical mixer.

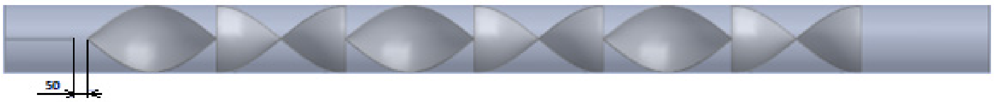

Figure 4: Computation model with a water inlet of 50 mm from the static mixer.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{v} \right) = S_m \qquad (1)$$

where ρ is the density, t is the time, and $\vec{v}$ is the velocity vector. On the right side of the equation is the source term S_m.

The law of conservation of momentum is expressed by the Navier-Stokes equations:

$$\frac{\partial \left(\rho \vec{v} \right)}{\partial t} + \nabla \cdot \left(\rho \vec{v} \vec{v} \right) = -\nabla p + \nabla \cdot \left(\overset{=}{\tau} \right) + \overrightarrow{S_m} \qquad (2)$$

where p is the static pressure, $\overset{=}{\tau}$ is the stress tensor and $\vec{S}_m$ are source terms (gravitational body force and external body forces).

The transport equation for the mass fraction of the species is defined as:

$$\frac{\partial \left(\rho Y_i \right)}{\partial t} + \frac{\partial \left(\rho \vec{v} Y_i \right)}{\partial x_j} = \frac{\partial}{\partial x_i} \vec{J_i} + R_i + S_i \qquad (3)$$

where Y_i is the local mass fraction of each species, J_i is the diffusion flux of species i, R_i is the net rate of production of species i by chemical reaction and S_i is the user-defined source.

In terms of physical properties, viscosity is defined for a highly viscous material using the Herschel-Bulkley model [5], which has the following formula:

$$\eta = \frac{\tau_0}{\dot{\gamma}} + k \cdot \dot{\gamma}^{n-1} \qquad (4)$$

Initial tangential tension τ_0 is 0.5 MPa, consistency index k is 2500 kg·s^{n-2}·m^{-1} and the parameter n is 0.165. $\dot{\gamma}$ (1·s^{-1}) is the shear rate. This notation is characteristic of the Herschel-Bulkley model for the definition of viscosity. Water is defined by density ρ= 998,2 kg·m^{-3} and viscosity η = 0.001 Pa·s.

2. Setting up the computational model and boundary conditions

The fluid flow of the mixture (working material and water) was defined as laminar, incompressible and stationary flow. Furthermore, flow without heat transfer in a general 3D geometry is considered.

The viscosity of a non-Newtonian substance is defined by the Herschley-Bulkley model. The physical properties of the working material and water are given in Table 1.

Table 1: Physical properties of individual materials.

	Density ρ	Viscosity η
	[kg·m^{-3}]	[Pa·s]
Water	998.2	0.001
Working material	1000	$\eta = \frac{\tau_0}{\dot{\gamma}} + k \cdot \dot{\gamma}^{n-1}$ $\tau_0 = 0.5\text{MPa}$ $k = 0.25\text{kg.s}^{n-2}.\text{m}^{-1}$ $n = 0.165$

Furthermore, boundary conditions were set at the individual boundaries of inlets and outlet (Figure 6).

The flow boundary conditions were defined by flow rates at the inlets to the pipe (entry of the working material into the annulus "Inlet", entry of water "Inlet water" into the working material in the pipe from the tube). Values of flow rates are given in Table 2. The outlet condition from the pipe ("Outlet") is free flow from the pipe.

Table 2: Values of inlet flow boundary conditions.

Helical mixer TR219x6,3					
	Qmax [kg·s^{-1}]	Q1 [kg·s^{-1}]	Q2 [kg·s^{-1}]	Q3 [kg·s^{-1}]	Qmin [kg·s^{-1}]
Inlet of working material	0.7083	0.5666	0.425	0.2833	0.1416
Inlet of water	0.0354	0.0283	0.0213	0.0142	0.007
Helical mixer TR114x3,6					
	Qmax [kg·s^{-1}]	Q1 [kg·s^{-1}]	Q2 [kg·s^{-1}]	Q3 [kg·s$_{-1}$]	Qmin [kg·s^{-1}]
Inlet of working material	0.2833	0.2266	0.17	0.1133	0.0556
Inlet of water	0.0142	0.0113	0.0085	0.0057	0.0028

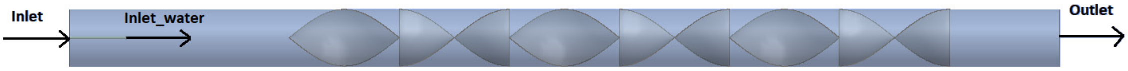

Figure 6: Boundary conditions of the computational model.

3. The results

The results of CFD numerical calculations are divided into two analyses. The first analysis is focused on evaluating the absolute pressure along the length of the pipe. Cuts were made in the pipe in which the pressure was evaluated. Subsequently, the total pressure loss in the pipe was evaluated.

The second analysis focused on the homogeneity of the mixture. Mixture homogeneity analysis is used in applications dealing with the mixing of materials to a certain quality. To determine this parameter, the coefficient of variation is used, which is the ratio of the standard deviation and the arithmetic mean.

The homogeneity is evaluated on several cross-sections of the pipe, and the concentration of the material in the given cross-section is determined to check the mixing of materials. A fully homogeneous mixture is when the mixture corresponds to the value of the parameter zero. For pipes with a static mixer, the homogeneity value is given depending on the length and diameter of the pipe. A different homogeneity is required for each application, which depends on the method of application of the given mixture. Homogeneity at the 0.005 level is required in most industrial applications. Higher homogeneity is required for special applications.

Ansys Fluent uses the uniformity index for this mixing case. The uniformity index represents the variability of the monitored field of the variable quantity on the given surface. In the case of a mixing pipe, the area is the cross-section of the pipe. This index can be expressed in two ways of evaluation. The first method is the surface-bound uniformity index. In this way, the variability of the instantaneous magnitude of the quantity can be determined. For example, in this case, concentration. The second method is a mass-based uniformity index that captures flow variability. The uniformity index determines the maximum homogeneity of the mixture with the number one. This distinguishes it from the coefficient of variation, which, on the other hand, determines a homogeneous mixture with the number zero. Calculation of the uniformity index of the surface bound index γ_a in the ANSYS Fluent environment is defined by:

$$\gamma_a = 1 - \frac{\sum_{i=1}^{n}\left[\left(\left|\phi_i - \bar{\phi}_a\right|\right)\cdot A_i\right]}{2\cdot\left|\bar{\phi}_a\right|\sum_{i=1}^{n} A_i} \tag{5}$$

where this computational relationship contains the monitored variable ϕ_i. In this case, it is the mass fraction of water. It also contains the average value of the variable $\bar{\phi}_a$ and area size A_i at the index of the surface element $\boldsymbol{i}$.

First, the variant with the pipe TR219x6.3 with the original water level was analyzed (Figure 2). It is apparent that the pressure losses are smaller as the flow rate decreases (Table 3).

Table 3: Pressure losses for individual mass flow rate of working material in the pipe TR219.

Q [kg·s⁻¹]	*Δp [MPa]*
0.7083	0.88
0.5666	0.87
0.425	0.85
0.2833	0.83
0.1416	0.77

Furthermore, the absolute pressure along the entire length of the pipe was evaluated. The result shows how significant the pressure loss in the pipe is caused by the mixer itself (Figure 7).

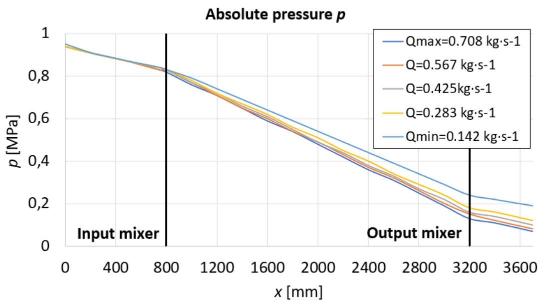

Figure 7: Absolute pressure along the length of the pipe TR219.

The subsequent evaluation is focused on the homogeneity of the mixed mixture, determined by the uniformity index γ_i. The limit value corresponding to the homogeneous mixture corresponds to the value γ_i =1. If the uniformity value reaches the value γ_i =0.995, the mixture is defined as fully homogeneous. The uniformity index for the helical mixer can be seen in the graph (Figure 8). The water inlet is at a distance of 206.5 mm from the beginning of the pipe. The uniformity index is zero at the input of the working material, since water is not present in the given area. Even before the entry of water, however, a mass fraction of water begins to appear in the given cross-section. After the water enters the pipe, the water immediately begins to mix, even before the mixture enters the mixer. This is because the mixture begins to be drawn into the mixer even before entering, and thus the water is mixed with the working material. After entering the mixer, the mixture is divided and mixed in the mixer, which shows the mass fraction of water in the graph (Figure 9). At maximum flow rate, the mixture is fully mixed in roughly half the length of the static mixer. It can also be noticed that the smaller the flow rate, the faster the mixing of the mixture occurs (Figure 8).

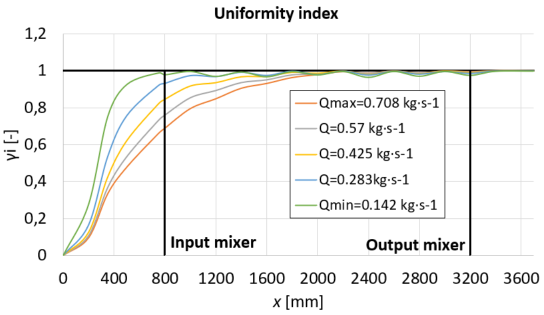

Figure 8: The uniformity index in the pipe TR219x6.3.

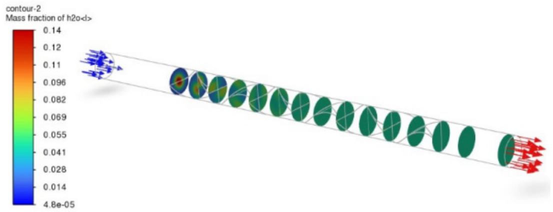

Figure 9: Illustration of the mixing process in a helical mixer using a mass fraction of water.

The evaluation was carried out similarly for the pipe TR114x3.6 (Figure 3). The pressure losses in this pipe are larger than in pipe TR219 (Table 4). The absolute pressure along the length of the pipe can also be seen (Figure 10). The homogeneity of the mixture for the pipe TR114 meets the conditions and the result is therefore like that of the pipe TR219 (Figure 11).

Table 4: Pressure losses for individual mass flow rate of working material in the pipe TR114.

Q [kg·s^{-1}]	*Δp [MPa]*
0.2833	0.97
0.2266	0.95
0.17	0.93
0.1133	0.91
0.0556	0.87

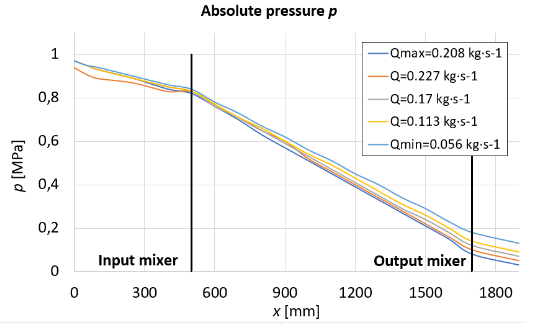

Figure 10: Absolute pressure along the length of the pipe TR114x3.6.

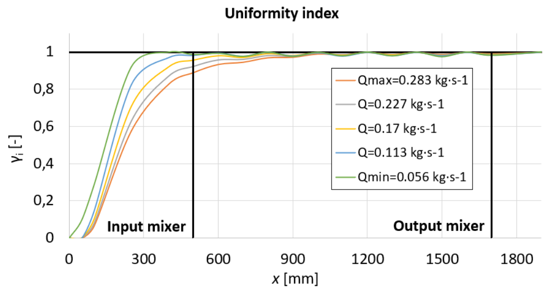

Figure 11: The uniformity index in the pipe TR114x3.6.

Another variant of the numerical calculation is the analysis of the flow of the mixture in the pipes TR219x6.3 and TR114x3.6 with a change in the position of the water inlet (Figure 4). Here, the homogeneity of the mixture and the influence of the displacement of the water inlet before the static mixer were investigated. The maximum and minimum flow rates of individual material were used. In the pipe TR219 the mixing process takes place mainly in the static mixer (Figure 12). The mixture is fully homogeneous at the end of the pipe and the application is therefore also suitable for this variant.

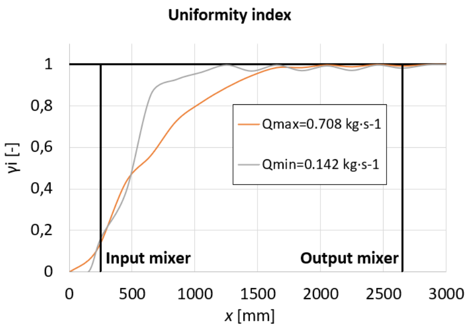

Figure 12: Uniformity index in pipe TR219 with different water inlet.

We see the same results with the pipe TR114 (Figure 13). Homogenization of the mixture already occurs during the flow through the mixer, at the maximum flow in the middle of the mixer. At minimum flow, full mixing occurs already at the beginning of the mixer.

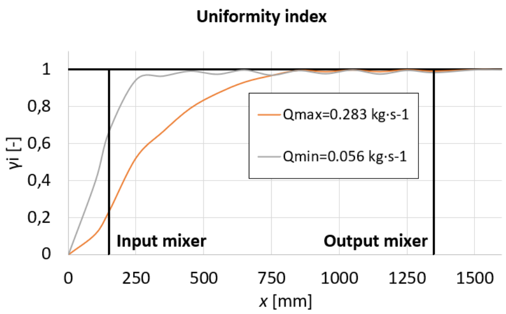

Figure 13 Uniformity index in pipe TR114 with different water inlet.

4. Conclusions

The paper discusses the CFD analysis of the numerical modeling of the mixing of a Newtonian substance with a highly viscous non-Newtonian substance in a pipe with helical static mixer. In the mathematical model, basic balance equations, equations for species and the Herschel-Bulkley model for defining the viscosity of a non-Newtonian substance were described. 3D models of individual geometry variants were created, including the computational mesh in the DesignModeler and Fluent Meshing programs. Different pipe clearances and the inlet position of the Newtonian substance were analyzed in a pipe model with a static mixer. Subsequently, the physical properties of individual substances and the types and parameters of boundary conditions of flowing substances were set in the ANSYS Fluent environment. As part of the post-processing after performing the numerical calculations of the 3D models, the results were mainly processed using excel graphs comparing individual variants from the evaluated data. The results are supplemented by the distribution of the mass fraction of water in individual sections along the length showing the process of homogenization of the mixture.

The absolute pressure and pressure loss in the pipe and the process of homogenization of the mixture were analyzed in detail. The homogeneity of the mixture was evaluated using the uniformity index. Two variants (different water inlet to the pipe before the static mixer) for different pipe diameters (TR219x6.3 and TR114x3.6) were compared. It is evident from the results that the required mixing of substances will occur in both variants. For use in the given problem, both variants are therefore for different pipe configurations, but due to smaller pressure losses, the TR219 pipe mixer is preferable. In both cases, there is sufficient mixing and thus the achievement of sufficient homogeneity of the mixture.

Acknowledgments

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