Acta Mechanica Slovaca **26 (2): 42- 47,** June **2022**
https://doi.org/10.21496/ams.2021.030

Acta Mechanica Slovaca
ISSN 1335-2393
www.actamechanica.sk

Methods of Linearization of Nonlinear Dynamic System

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Abstract: In This Research paper, described different linearization methods in terms of analytical and numerical simulation (Computer added linearized -FE) method and Digital image correlation method for prediction of loading analysis. Article can be helping for linearization in nonlinearity of systems, that what is using different methods likes Numerical (approximation), Analytical (Exact) and Experimental. Where, analytical is difficult to calculate in computers so it uses numerical methods. That is nearer to exact values so we can determine behaviour of dynamical systems. Numerical simulation is static with different time, so analysis is easy for its. Numerical methods work on algorithms, so it is easy to evaluate for calculating in computers. For future aspects image correlation method is more helpful to determine behaviour of nonlinear dynamic system which will discretize system with different displacement and FEM analyses to dynamically nonlinear system.

Keywords: FEM; Boundary finite element method; exact methods; approximation methods; Lagrange Rayleigh Ritz technique and Monte Carlo simulation; Image correlation methods; Discretization; harmonic balance & static method; Eigen value; Optimal linear and Jacobean linearization methods; Linearized Finite element static energy analysis; Static energy simulatio

1. Introduction

Linearization: The Technique, which will help to understand non-linear dynamic system and Evaluate them for now in industry revolution 4.0, methods are being changed by time with ease. Approximate nonlinear differential strategy with differential condition for small excursion about balance. For linearization system variable change with other variable in finite way, on the opponent side for understanding of non-linear dynamic system which shows variations in infinite way, such as velocity, acceleration or differential with other variables, for making it in to linearized. It supposed to function of variable to be zero. Even more nonlinear dynamic system can be balanced or homogenized for linearity to understand system, even this method has too many scopes in artificial intelligent robots to predict system itself and manipulate themselves with ergonomics of features. The analytical calculations seem to reliable results but for industry 4.0 revolution is about to use numerical calculation. Even if it is having some efficient methods like as boundary finite elements method (SBFEM) and most trended method is Finite element method. More ever method can be hybridized with analytical and numerical which is using built in function for nonlinear system which excited with external-internal energy likewise harmonic and random loading energy, vibration to energized system in to dynamical would be having solution with harmonic balance method and static method to analytical solved but in numerically we can using finite

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element static method which carried out from review of Lagrange Rayleigh Ritz technique and Monte Carlo analysis simulation. Linearization takes place by its applied external – internal energy or vibration [1-2].

And nonlinear Finite element method FEM analysis also can be linearized which employed Newton-Raphson method. This approach is FE approximation methods are included in weak draft. Draft helps us for semi- discrete formulation to acquire matrix. Other approach is using of linearized weak then we can get weak draft, linearization possible via gateaux derivative, after linearized weak draft is helped from FE approximation to be linearized fem analysis [8-9].

System vibration can be executed using of different computation methods. Where Fem can be specifying more constrains with motilities. Then only it is leading complexity which will give low accuracy approximation with high threshold. Static energy method will help to achieve approximate nearer accuracy in Dynamic system. Static energy method will be averaging approximation of system response static energy is making it behave in equilibrium system [3-6].

Optimal linear and Jacobean linearization methods are for analytical calculation. Optimal linear is to generate linearization in nonlinear system even more Jacobean is supposed to manipulate dynamical system therefor using of both methods can be identified nonlinear dynamic behaviour [4].

2. Analytical Draft

When creating, exciting, analysing, displaying and evaluating complicated results multidisciplinary systems we use computer support with algorithmic procedures, but it is crucial to have the skills and experience to correctly interpret the results obtained and then purposefully change the parameters of the system to achieve the required dynamic. Properties of mechanical systems and control systems already existing, but mainly new products. Although explicit mathematical models can be compiled and solved in a closed form only for dynamic systems with one degree of freedom of movement, the acquired knowledge is the key to understanding the more complex dynamic properties of systems with multiple degrees of freedom movement. The advantage is that mathematical models of dynamic systems and control systems they contain the same equations of motion, which allows the use of analysis techniques and the results obtained effectively apply from one area to another. Analytical approach thus suitably depicts the physical relationship of the system and at the same time deeply analyses the problem area [4-16].

2.1. Nonlinear system Optimized to linear model [4]

Assumption of nonlinear system from equation

$$\frac{dx}{dt} = F_1(x(t)) + F_2(x(t)).v(t) \tag{1}$$

For optimizations

$$F_1(x') + F_2(x')v = C_1x' + C_2v \tag{2}$$

Lagrange constant

$$(\lambda) = \frac{[(x')^T \Delta F(x') - F(x')]}{\|x'\|} \tag{3}$$

Assuming Nonlinear dynamic formulations

$$(m+M)\ddot{x} - ml\dot{\theta}\sin\theta + ml\ddot{\theta}\cos\theta + f.\dot{x} = F \tag{4}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{I+ml^2}{(m+M)I+mMl^2} \\ 0 \\ -\dfrac{ml\cos x_3}{mI+M(I+ml^2)} \end{bmatrix} v + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-f_r(I+ml^2)}{(m+M)I+mMl^2} & m^2l^2g\dfrac{\cos x_3}{(m+M)I+mMl^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{mlf_r\cos x_3}{mI+M(I+ml^2)} & \dfrac{-m^2-Mmgl}{mI+M(I+ml^2)} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{5}$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a & b(\cos x_3 - x_3 \sin x_3) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & d\cos x_3 & e - d.x_2 \sin x_3 & 0 \end{bmatrix} \quad \text{and} \quad C_2 = \begin{bmatrix} 0 \\ c \\ 0 \\ h\cos x_3 \end{bmatrix} \tag{6}$$

3. Computational software method

Stability of image in controlling points are given by Eigen value of exponential system. Which generate control matrix to be linearized. Forward-Differential Approximation FDA Method and Centre Difference Approximation CDA method are numerical linearization methods.

When, dynamic system creates linear form from equilibrium point. The linear form response in small distribution which shows variable depend time on small distribution of variable. Whose relation describes matrix A and it is small distribution. And feedback of v is assumed. FDA and CDA are supposed to nonlinear feedback system of algebraic variable [5-7].

3.1. Numerical Methods

Forward Differential approximation method is assessed with equilibrium control. New technology image correlation will capture system behaviour and applying material constrains and forces as dynamical system will generate system simulation to understand systems. In, numerical simulation is taking place static system behaviour and apply algorithm to connected body. More even better understanding system is discretized and simulate. Moreover, similar as FEM analysis which is to generate model system is main difference between them. High Speed capture cameras are needed to generate model behaviours [12-13].

Linearized finite element statistical analysis generating constructive system with nonlinear joints are activated with harmonic loading and random loading. As per loading nature system gets linearized. Statistical (uneven) and harmonic natural loading can be processed by harmonic Equilibrium and static Equilibrium as per arranging order. Constructive vibrational system leads by these loading and can be investigated by verity of computational methods mostly is used numerical FE Method (Approximate) which and manipulate mobility of system. For accurate result of approximation FEM, it is hybridized with static energy simulation to remove drawback of approximation value. Combine FEM and statistical analysis will get to achieve exact value of dynamic system [8]. Energy balancing phenomena will be applied hereby averaging of iterative values at specific points with the help of statistical energy examination. Vibrational dynamic system proposed FE geometrical nonlinearity with assessment. Small and moderate quantity of nature of system will be assessed by using of Lagrange-Rayleigh-Ritz method. Even large nature system will be assessed with FE statistical energy simulation. Harmonic and statistical equilibrium is with finite elements static energy simulation is trying to reach exact solution for linearization [10-11]. Forward Difference approximation and Centre difference approximation are numerical methods. Forward method A-matrix is written [5].

$$h.A = \frac{dx}{dt} \tag{7}$$

Size of h is directly proportional to assessment. Eigen values are consequence and terms are of A matrix.

Centre difference approximation is numerically Forward approximation [5].

$$h.A = \frac{d(x_1 - x_2)}{dt} \tag{8}$$

Where, $x_1 = x$ is (x_i^+)

$x_2 = x$ is (x_i^-)

Difference of value of this FDA is perturbation value. Therefore, poor signal system is assessed value of perturbation is poor therefore it is point linearity. FDA and CDA is efficient method to high degree of linearization. FDA is achieving high effectiveness than CDA. From overall perturbation value is affected highly by answered Eigen assessment. For higher perturbation, CDA is approximately nearer to Analytical method. FDA is more efficient method in this both differences [5-9]. Working Robot is dynamic nonlinear system. This nonlinear dynamic system to control it is need linear system for controlling of linear application PID is mainly using. Aspects can analysis and synthesis of nonlinear controller system. It has to, Krasovskii's stability if asymptotic equilibrium theorem for nonlinear system [14-15].

3.2. Harmonic Equilibrium

Sinusoid with amplitude of nonlinear system will include to linearization in harmonic equilibrium.

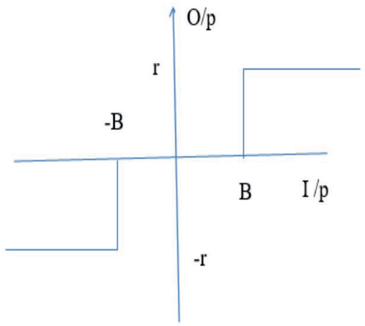

Figure 1: Harmonic graph

4. Simulation

Spring mass system is evaluated in MSC ADAMS simulation software which is computed by using numerical methods.

Consider spring mass nonlinear dynamic system, mass = 100 kg; spring k = 6 N/m; c = 0.07 Ns/m.

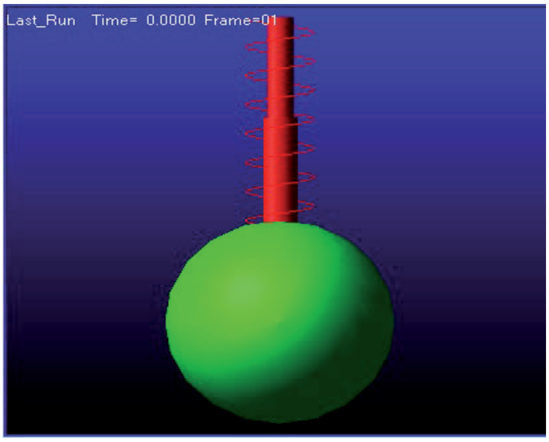

Figure 2: Spring mass

4.1. Simulation results

Body is accounted as Forces and Torque in x, y, z direction in order. It is postprocessor of system simulation in following figures 3-9.

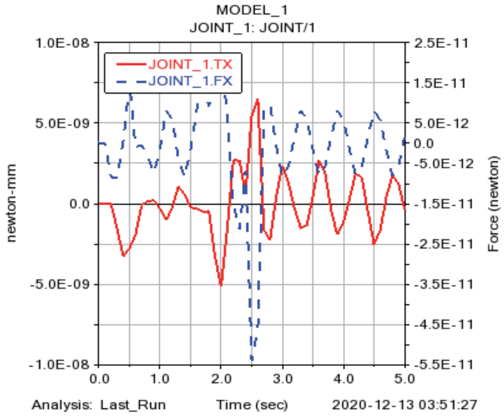

Figure 3 F & T in x system coordinate

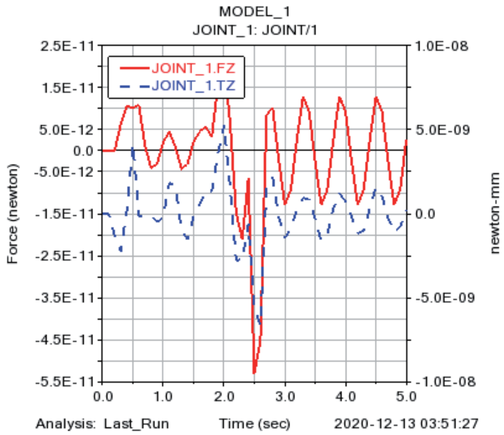

Figure 4: F & T in z system coordinate

From result of numerical simulation method will presents Forces and Torque behaviour of nonlinear dynamic system (spring-mass). Postprocess will presents results in different prospects which generalised by categories.

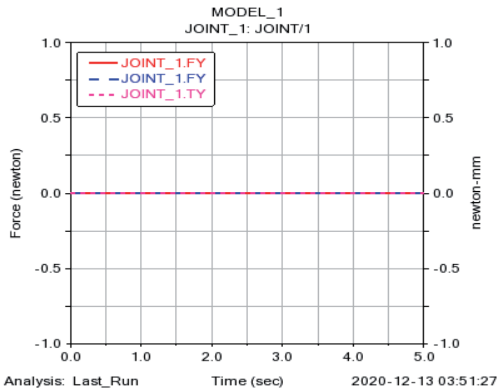

Figure 5: F & T in y system coordinate

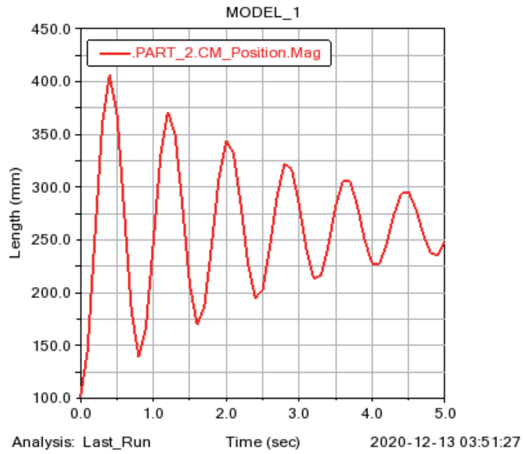

Figure 6: Position mag system simulation

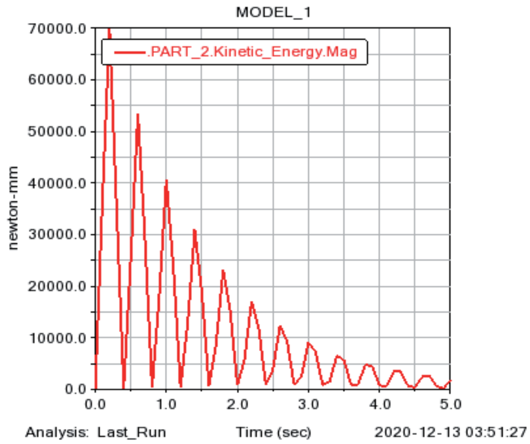

Figure 7: Kinematic mag system

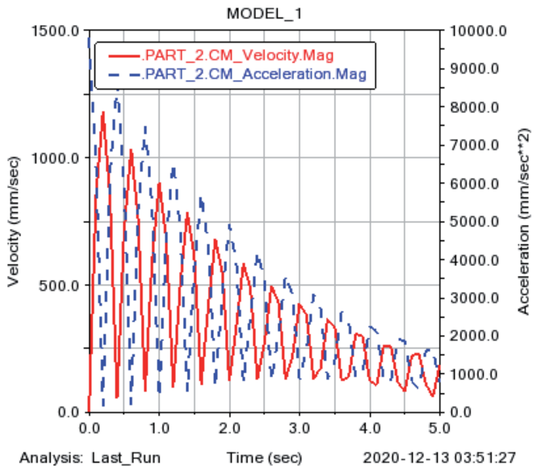

Figure 8: Velocity mag system simulation

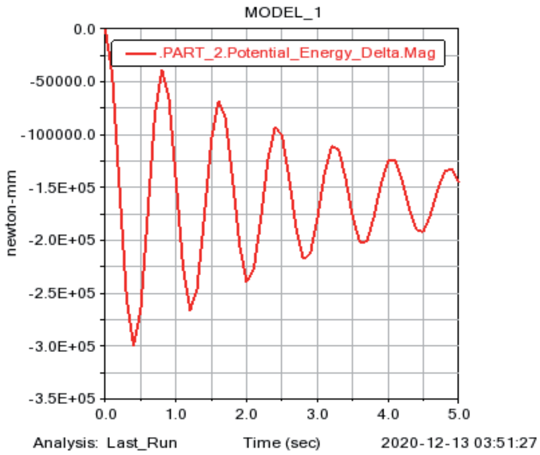

Figure 9: Potential Energy system simulation

5. Conclusion

Referring articles and numerical study this paper sum up with numerical method of linearization is efficient and time effective study. In actually it is statically visualized position of changes in behaviour of model. Is discretize body and make them to static in specific position, numerical algorithm will compute speedy then analytical method where approximation iteration algorithm is efficient then direct method of simulation, in direct method it is generating matrix and Eigen value which consume time than algorithmic method. Finite element method is most nearer than other because it is discretized in tetrahedral and basic element which specified partial differential equation which connectivity transfer to other one and present exact result if we more discretized. Where analytical is time consuming method. Analytical model is giving exact value but it is not suitable in application due to time consuming and so on. From all over assassination it is conclude that FEM Analysis of linearization is suitable than other linearization and FEM with image correction method is mostly suitable for future work and effectiveness of simulation than other linearization method.

Acknowledgments

The authors would like to thank to Slovak Grant Agency project VEGA 1/0389/18, grant project KEGA 018 TUKE-4/2018, grant project KEGA 030 TUKE-4/2020 supported by the Ministry of education of Slovak Republic.

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