Acta Mechanica Slovaca
ISSN 1335-2393
www.actamechanica.sk

Positioning Measurements of Two Industrial Robots

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Abstract: This paper deals with the positioning accuracies of two industrial robots, i.e., a KUKA KR 15/2 and a Fanuc LR Mate 200iC comparing measurement and modeling. The accuracy measurements are performed by Renishaw XL-80 type laser measurement system. Both robots are Articulated Manipulator (RRR), and their descriptions of Denavit-Hartenberg parameters are very similar. The inverse kinematics of the two industrial robots are also solved and analytic formulae are derived. The measurements are carried out along two perpendicular straight lines and the results are compared.

Keywords: Robot, simulation, measurement, inverse kinematics, Denavit-Hartenberg parameters.

1. Introduction

Nowadays robots are used even for humanoid purposes [1] in addition to industrial ones, e.g., assembly, painting, welding, workpieces handling and machining. The positioning accuracy of the robots in the later case, i.e., in machining, is a crucial question [2].

The Institute of Machine Tools and Mechatronics, in University of Miskolc, have two industrial robots, one of them is a 20 years old KUKA KR 15/2 robot and the other one is a 5 years old Fanuc LR Mate 200iC robot. Before their application to machining, it is advisable to check the accuracy of positioning. One of the best measurement device is the laser measurement system, such method was also used in [3]. This optical method is well applicable for measurements along straight line motions.

In Section 2 the kinematical description with the Denavit-Hartenberg (DH) parameters and the inverse kinematics are given for the two industrial robots ([4], [5], [6]). Measurements with laser measurement system are detailed and demonstrated in Section 3 ([7], [8]). Some concluding remarks are drawn in Section 4.

2. Kinematical description of the robots

The two investigated industrial robots have very similar structures and both of them can be analyzed by DH description. According to [6] there are four DH parameters, i.e., the link length a_k, link twist α_k, link offset s_k and joint angle θ_k, which are defined as follows:

- a_k *is the distance between the axis* z_{k-1} *and* z_k, *and is measured along the axis* x_k
- a_k *is the angle between the axis* z_{k-1} *and* z_k
- s_k *is the distance from origin* o_{k-1} *to the intersection of the* x_k *axis with* z_{k-1} *measured*

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along the z_{k-1} *axis*

■ θ_k *is the angle from* x_{k-1} *to* x_k *in a plane normal to* z_{k-1}

The applied coordinate systems for the robots, i.e., KUKA KR 15/2 and Fanuc LR Mate 200iC are shown in Fig. 1a and Fig. 1b, respectively, and the DH parameters are listed in Table 1 based on [4] and [5].

The following homogenous transformation can be written between two consecutive $(k-1,k)$ coordinate systems:

$$\boldsymbol{H}_{k-1,k} = \begin{bmatrix} \boldsymbol{h}_{k-1,k} & \boldsymbol{r} \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_k & -s\theta_k\, c\alpha_k & c\theta_k\, s\alpha_k & a_k\, c\theta_k \\ \theta_k & c\theta_k\, c\alpha_k & -c\theta_k\, s\alpha_k & a_k\, s\theta_k \\ 0 & s\alpha_k & c\alpha_k & s_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where $s\theta_k$, $c\theta_k$ and $s\alpha_k$, $c\alpha_k$ denotes sine and cosine of θ_k and α_k, respectively.

Table 1: *DH parameters of robots KUKA KR 15/2 and Fanuc LR Mate 200iC.*

Parameters / Joints	s_k [mm] KUKA	s_k [mm] Fanuc	θ_k [°] KUKA	θ_k [°] Fanuc	α_k [°] KUKA	α_k [°] Fanuc	a_k [mm] KUKA	a_k [mm] Fanuc
J1	675	330	0°	0°	+90°	+90°	300	75
J2	0	0	+180°	+90°	0°	0°	650	300
J3	0	0	0°	0°	+90°	+90°	155	75
J4	600	320	0°	0°	-90°	-90°	0	0
J5	0	0	0°	0°	+90°	+90°	0	0
J6	140	140	0°	0°	0°	0°	0	0

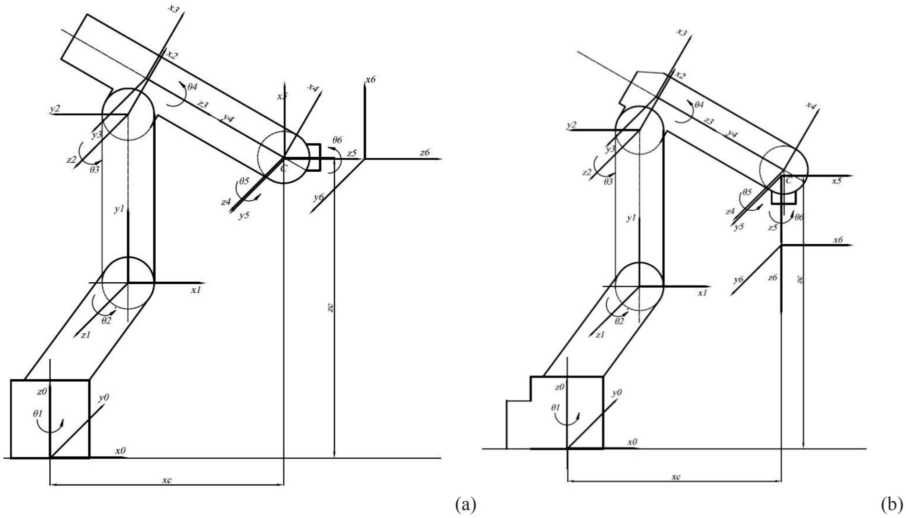

Fig. 1: *Coordinate systems of (a) KUKA robot and (b) Fanuc robot.*

The inverse kinematics of the robots are analyzed in order to determine the accurate values of the joint angles (θ_k) in measured points along prescribed straight lines. It is supposed that the orientation and path of the end effector are given. The angles of the first 3 joints give the position (x_c, y_c, z_c) of the end effector, while the other 3 angles of the joints determine orientation of the end effector.

The angle θ_1 is shown in Fig. 2, its value can be obtained by the help of x_c and y_c:

$$\theta_1 = \arctan 2\,(y_c\,,\,x_c) \quad (2)$$

where $\arctan2$ provides values in interval $(-\pi,+\pi)$.

Joint angles θ_2 and θ_3 are shown in Fig. 3.

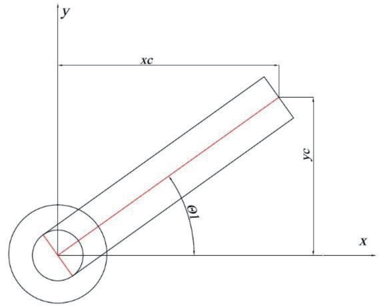

Fig. 2: *Over view of the robot.*

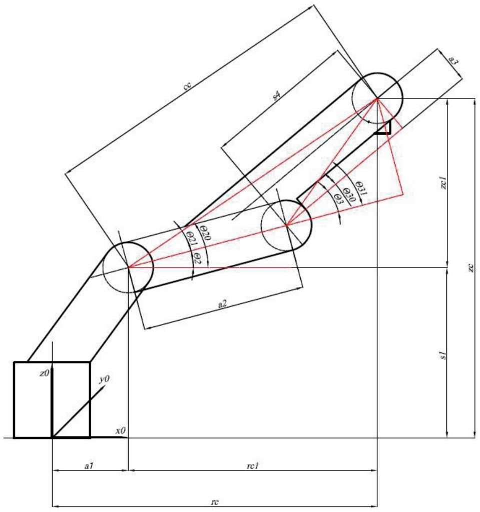

Fig. 3: *Side view of the robot showing θ_2 and θ_3.*

Distance parameters a_1, a_2, a_3, s_1, s_4, z_c, r_c, z_{c1} and r_{c1} shown in Fig. 3 are given while the last of them can be determined as follows:

$$r_c = \sqrt{x_c^2 + y_c^2} \tag{3}$$

$$z_{c1} = z_c - s_1 \tag{4}$$

$$r_{c1} = r_c - a_1 \tag{5}$$

The joint angle θ_2 and θ_3 can be also obtained from simple trigonometry [6] using cosine theorem:

$$\theta_3 = \theta_{30} - \theta_{31} = \arctan2\left(\pm\sqrt{1-D^2}\ , D\right) - \arctan2(a_3, s_4) \tag{6}$$

where

$$D = \frac{cc^2 - a_2^2 - \left(\sqrt{s_4^2 + a_3^2}\right)^2}{2 \cdot a_2 \cdot \sqrt{s_4^2 + a_3^2} \cdot \cos(180^\circ - \theta_{30})} \tag{7}$$

and

$$\begin{aligned} \theta_2 &= \theta_{20} - \theta_{21} \\ &= \arctan2\left(zc - s_1, \sqrt{x_c^2 + y_c^2} - a_1\right) \\ &\quad -\arctan2\left(\begin{array}{l}\sqrt{s_4^2 + a_3^2} \cdot \sin(\theta_{30}), \\ a_2 + \sqrt{s_4^2 + a_3^2} \cdot \cos(\theta_{30})\end{array}\right) \end{aligned} \tag{8}$$

where

$$\theta_{30} = \arccos(D) = \arctan2\left(\pm\sqrt{1-D^2}\ , D\right) \tag{9}$$

Joint angles θ_4, θ_5 and θ_6 can be obtained using transformation matrix $\boldsymbol{R}$ given by Euler angles assuming downward vertical orientation of the end effector as shown in Fig. 1b:

$$\boldsymbol{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{10}$$

Transformation matrix $\boldsymbol{R}$ in (9) is equal to $\boldsymbol{h}_{0,6}$ matrix which is a portion of the $\boldsymbol{H}_{0,6}$ homogoneus transformation matrix [6]:

$$\boldsymbol{H}_{0,6} = \begin{bmatrix} \boldsymbol{h}_{0,6} & & & \boldsymbol{r} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

The transformation matrix $\boldsymbol{h}_{3,6}$ between the 3rd and 6th coordinate systems can be expressed as follows:

$$\boldsymbol{h}_{3,6} = (\boldsymbol{h}_{0,3})^T \cdot \boldsymbol{R} = \begin{bmatrix} \cos(\theta_1)\cdot\cos(\theta_2+\theta_3) & -\sin(\theta_1)\cdot\cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3) \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ \cos(\theta_1)\cdot\sin(\theta_2+\theta_3) & -\sin(\theta_1)\cdot\sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) \end{bmatrix} \tag{12}$$

where

$$(\boldsymbol{h}_{0,3})^T = \begin{bmatrix} \cos(\theta_1)\cdot\cos(\theta_2+\theta_3) & \sin(\theta_1)\cdot\cos(\theta_2+\theta_3) & \sin(\theta_2+\theta_3) \\ \sin(\theta_1) & -\cos(\theta_1) & 0 \\ \cos(\theta_1)\cdot\sin(\theta_2+\theta_3) & \sin(\theta_1)\cdot\sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) \end{bmatrix} \tag{13}$$

The transformation matrix $\boldsymbol{h}_{3,6}$ in (12) can be also defined by the multiplication of three consecutive relative transformation matrices:

$$h_{3,6} = \prod_{i=4}^{6} {}_{i-1,i} = \begin{bmatrix} c(\theta_4)\cdot c(\theta_5)\cdot c(\theta_6) - s(\theta_4)\cdot s(\theta_6) & -c(\theta_4)\cdot c(\theta_5)\cdot s(\theta_6) - s(\theta_4)\cdot c(\theta_6) & c(\theta_4)\cdot s(\theta_5) \\ s(\theta_4)\cdot c(\theta_5)\cdot c(\theta_6) + c(\theta_4)\cdot s(\theta_6) & -s(\theta_4)\cdot c(\theta_5)\cdot s(\theta_6) + c(\theta_4)\cdot c(\theta_6) & s(\theta_4)\cdot s(\theta_5) \\ -s(\theta_5)\cdot c(\theta_6) & s(\theta_5)\cdot s(\theta_6) & c(\theta_5) \end{bmatrix} \quad (14)$$

Comparing the 3rd columns and 3rd rows of (12) and (14) one can obtained the following three equations:

$$\cos(\theta_5) = \cos(\theta_2 + \theta_3) \quad (15)$$

$$\sin(\theta_4)\cdot\sin(\theta_5) = 0 \quad (16)$$

$$\frac{\sin(\theta_5)\cdot\sin(\theta_6)}{-\sin(\theta_5)\cdot\cos(\theta_6)} = \frac{-\sin(\theta_1)\cdot\sin(\theta_2+\theta_3)}{\cos(\theta_1)\cdot\sin(\theta_2+\theta_3)} \quad (17)$$

and the solutions for θ_4, θ_5 and θ_6:

$$\theta_4 = 0 \quad (18)$$

$$\theta_5 = \theta_2 + \theta_3 \quad (19)$$

$$\theta_6 = \theta_1 \quad (20)$$

3. Measurement of the positioning

Fig. 4a shows the measurement setup for the KUKA robot while Fig. 4b illustrates the measurement setup for the Fanuc robot, where No. 1 is the linear reflector, No. 2 is the linear interferometer with linear reflector and No. 3 is the laser head ([7]). The linear reflector denoted by No. 1 is moved by the end effector, while the interferometer No. 2 and laser head No. 3 are in a static positions. The resolution of the system is 0.001µm [8].

Due to technical reasons both in the starting position and during the measurement the end effector of the KUKA robot is kept in horizontal direction, while in case of Fanuc robot the end effector is kept in vertical direction, as it is shown in Fig. 1 and Fig. 4. The joint angles of the robots in the starting position are given in Table 2.

During the measurements of the robots the end effectors are moved horizontally firstly along X direction then secondly along Y direction with ±150mm from the starting positions. In both cases the distance steps are 1mm and the velocity of the motions was prescribed to 25 mm/s. The positions were measured by laser head and the errors are

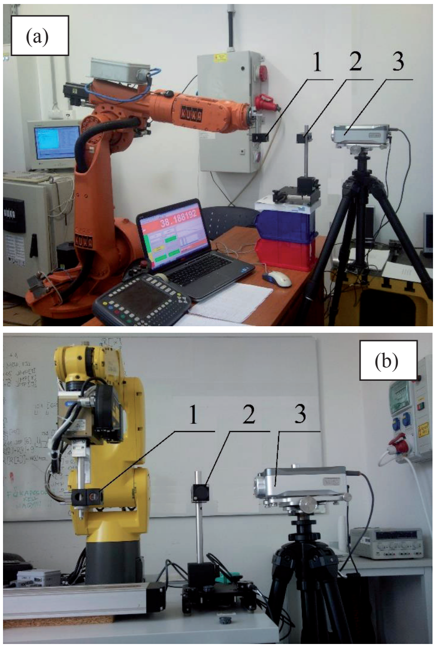

Fig. 4: *Positioning measurement: (a) KUKA robot and (b) Fanuc robot.*

Table 2: *Starting joint angles of robots KUKA KR 15/2 and Fanuc LR Mate 200iC.*

Joint	KUKA Joint angle[°]	Fanuc Joint angle[°]
J1	-90°	0°
J2	-90°	0°
J3	95°	-14°
J4	0°	0°
J5	-5°	-76°
J6	0°	90°

shown in Fig. 5 and Fig. 6 for motions in X and Y directions, respectively.

The positioning errors of the KUKA robot are denoted by green lines, while errors of the Fanuc robot are denoted by blue ones. In Fig. 5 and Fig. 6, the measurements in negative and in positive directions are given in a/ and b/ diagrams, respectively.

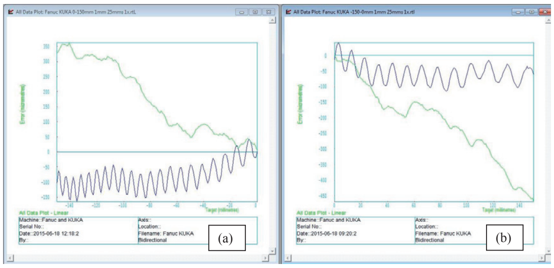

Fig. 5: *Positioning errors (a) in negative X direction, (b) in positive X direction, where green and blue lines denote the KUKA and Fanuc robots, respectively.*

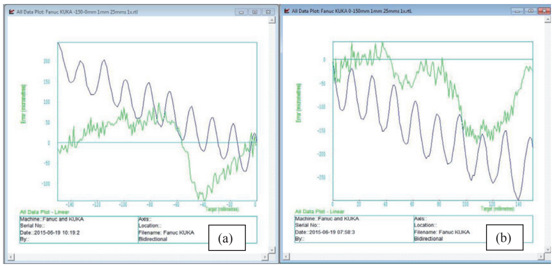

Fig. 6: *Positioning errors (a) in negative Y direction (b) in positive Y direction, where green and blue lines denote the KUKA and Fanuc robots, respectively.*

One can see that the positioning errors show increasing tendencies during the motion from the starting point in ± X directions (see Fig. 5). On the measured interval (-150mm, +150mm) the KUKA robot performed about +350µm and -450µm maximal errors, while the Fanuc robot -150µm and -100µm errors. The measurements in Y directions are shown in Fig. 6 where the KUKA robot produced about -150µm maximal errors, while the Fanuc robot +250µm and -300µm errors. The diagrams for the KUKA show random oscillations, and almost regular oscillations can be seen for the Fanuc robot.

The above measurements were performed for different velocities and distance steps, and similar inaccuracies were obtained. Due to lack of space results are not shown here.

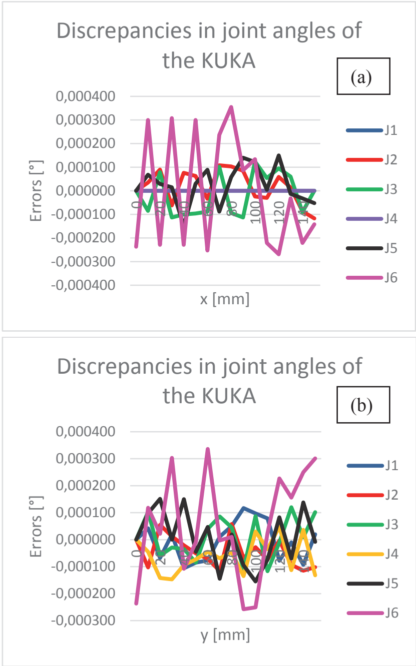

Fig. 7: *Discrepancies in joint angles of KUKA robot comparing values provided by the controller and the theoretical formulae during the motions are (a) in X direction (b) in Y direction.*

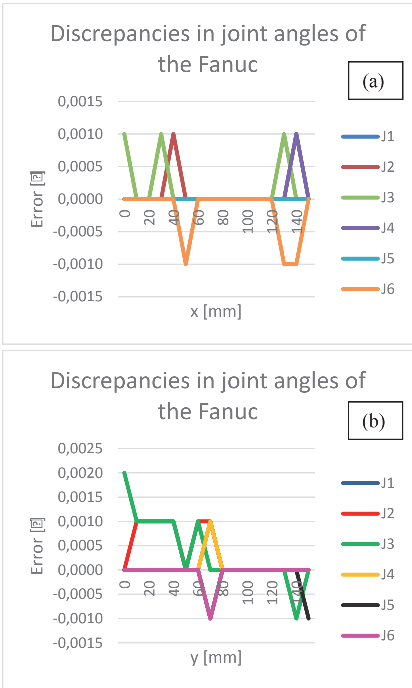

Fig. 8: *Discrepancies in joint angles of Fanuc robot comparing values provided by the controller and the theoretical formulae during the motions are (a) in X direction (b) in Y direction.*

In order to investigate the sources of the positioning errors, the theoretical values of the joint angles are compared to the data provided by teaching boxes of the robots. The discrepancies of the two values are shown in Fig. 7 and Fig. 8 for the KUKA and Fanuc robots, respectively. The data of joint angles are given with six decimal precision by the controller of KUKA robot and three decimal precision by Fanuc robot. Results show that the errors in angles are practically negligible, since it may cause errors in positioning only a couple of µm.

It is likely that the lion part of the positioning errors shown in Fig. 5 and Fig. 6 is resulted from the flexibility of the driving systems of the joints, which requires further investigation.

4. Conclusions

Positioning accuracies of two industrial robots have been analyzed in this paper. The positions of a robot KUKA KR 15/2 and a robot Fanuc LR Mate 200iC have been measured with Renishaw XL-80 type laser measurement system. The joint angles were evaluated by the help of analytical formulae, which were compared to angle values provided by the robot controllers. It was found that the discrepancies in joint angles are negligible.

It is shown that the positioning errors for the KUKA robot oscillate randomly, while almost regular oscillations can be seen for the Fanuc robot. Since the errors larger than 0.1mm in positioning the investigated robots are not applicable for precise machining.

Further investigations are needed to determine the causes of the positioning errors.

5. Acknowledgements

This research was carried out in the framework of the Center of Excellence of Mechatronics on Logistics at University of Miskolc.

6. References and notes

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