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Cyclic Calculation of Inverse Kinematics for n-link Serial Mechanism

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BIOGRAPHICAL NOTES

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KEY WORDS

Inverse Kinematics, Cyclic Coordinate Descent, Serial Chain Robot

ABSTRACT

Inverse kinematics of mechanic system is area of solution at many universities and scientific research institutions. This paper dealt with of solution problem of inverse kinematic by a cyclic coordinate descent method (CCD). This method allows use n-link chain mechanism without complicated changing of control algorithm. The article is focuses on the issue of setting the number of elements and also presents a modified method the incremental CCD, which is compared with this method

1. Inverse kinematic problem

Inverse kinematic problem can be described as „finding" the right parameters for each kinematic joint of mechanism to achieve the desired result in a pre-defined position of member. The available literature provides several definitions for this problem. In defined shape of mechanism, Fig. 1, we know where its base is. With the control system and sensors know the current mechanism shape and position of the end point of the mechanism. If we define a new position in which to get the mechanism end point, for the computation algorithm, the actual end point position is starting position and its task is to calculate values for individual joints q_1 to q_n, so the end point has the desired position with reasoning error limit [1].

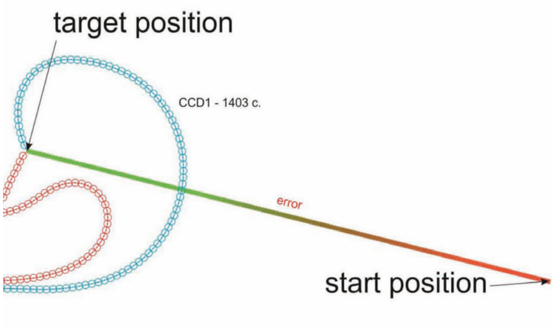

Fig. 1: *New rope without primary and subsequent lubrication.*

There are several computation methods that are described in available literature. Whatever the method using Jacobian and their subsequent modifications [1,3,4], or other methods [2]. The biggest problem with these methods arises with the redundant manipulators, when the desired position is possible to achieve by the infinity combinations of q_i values.

2. Cyclic computation

This method allows computing the inverse kinematics of redundant mechanisms with principle of gradual calculation parameters for each kinematics joint separately.

2.1 Cyclic coordinate descent

In this method there is gradual calculation for each kinematic joint separately from the last joint in a series of mechanism to the first. This cycle is repeated until it reaches desired position of the end point of mechanism respectively until value of error isn't smaller then error limit. The q value is a calculated difference between actual angle and desired angle. This q value can be limited by the maximum value of step size. The algorithm of calculation in basic form for one "finding" point with defined max step size and without optimization algorithm has form:

```
If (error > errorLimit)
cycle = 0
  For i = n : 1
  Find vector between the i-joint and end point
  Find vector between the i-joint and desired point
  Identify different between these vectors
      If (difference > maxStep)
  difference = maxStep
  end
  Change angle of i-joint for difference value
  Recalculate position of all joint from i to n
  cycle = cycle +1;
      Find error between end point and desired
point
  If (error < errorLimit)
  End cycle For
  Else
  i = i – 1
  if i == 0
  i = n
  end
  end
  end
end
```

2.2 Incremental cyclic coordinate descent

This method allows with last link "follow" desired point. Procedure of computation is in the next algorithm. The algorithm is for one "finding" point with defined a max step size and without an optimized algorithms. Difference between CCD and iCCD is marked:

```
While (error > errorLimit)
  g = 0
  cycle = 0
  For i = n : 1
      Find vector between the i-joint and end point
      Find vector between the i-joint and desired point
      Identify different between these vector
          If (difference > maxStep)
             difference = maxStep
          end
      Change angle of i-joint for difference value
      Recalculate position of all joint from i to n
      cycle = cycle +1;
      Find error between end point and desired point
      If (error < errorLimit)
          End of cycle For
      Else
          i=i-1
          If i == g or i == 0
             I = n
             g = g - 1
```

```
            If g == -1
                  g = n - 2
            end
        end
    end
  end
end
```

3. Mathematical model

Between a main criteria for choose the right inverse kinematics algorithm belongs a number of calculation cycles to an achieve desired task and a time of computation. Computation times are changed depending on the type of computer on which the calculation is performed. Therefore, all calculations for this post were taken on a single computer. This part of article presents obtained results for a chain link mechanism composted from of n = 10 to 100 links with step 10 for changing maximum step size of i-joint angle - 1°, 10° a 100°.

Starting position of end point is xp = n a yp = 0 and desired position of the mechanism is defined as follows:

1. point: $x_k = n/2$ a $y_k = n/2$,
2. point: $x_k = 0$ a $y_k = n/2$,
3. point: $x_k = -n/2$ a $y_k = n/2$.

Link n has unit length and error limit is set to 0,01. The computation was made in application programmed in MATLAB©.

3.1 Cyclic coordinate descent

Fig. 2 a 3 shows graphs of times for an individual computation with CCD method. The computation of a third point and step size set to 100°, the number of links 30 and more. The mechanism has not reached the desired point after n*10000 cycles. For this reason there are zero values in the table, Tab. 1. Same problem occurred with the third point and the movement on the track with a max angle step 1° and a number of links 50, Tab. 2.

Table 1: *Times for calculating the point with CCD method.*

	10	20	30	40	50	60	70	80	90	100
Point 1 1°	0,053	0,078	0,138	0,215	0,298	0,420	0,499	0,748	1,052	1,142
Point 1 10°	0,008	0,008	0,139	0,170	0,230	0,370	2,638	3,971	7,891	10,424
Point 1 100°	0,049	0,086	0,591	0,826	1,687	2,382	4,028	9,662	17,553	28,729
Point 2 1°	0,042	0,092	0,155	0,247	0,343	0,473	0,432	0,651	0,834	0,733
Point 2 10°	0,021	0,041	0,224	0,103	2,155	0,651	1,297	0,565	6,750	11,225
Point 2 100°	0,071	0,337	1,627	4,520	11,462	21,296	44,503	68,511	99,487	156,678
Point 3 1°	0,121	0,297	0,541	0,858	10,022	15,316	25,025	30,228	54,323	68,645
Point 3 10°	0,078	0,507	2,456	4,059	9,947	1,085	16,573	16,910	45,592	10,378
Point 3 100°	17,587	64,037	139,113	0,000	0,000	0,000	0,000	0,000	0,000	0,000

Table 2: *Times for calculation desired point on the path in the method CCD.*

time [s]	10	20	30	40	50	60	70	80	90	100
Path 1 1°	0,136	0,536	2,400	6,400	8,830	27,562	47,782	81,993	126,907	182,757
Path 1 10°	0,067	0,457	1,882	2,572	5,320	10,792	42,308	61,188	83,604	113,460
Path 1 100°	0,079	0,571	2,278	5,788	11,464	19,584	30,981	46,562	66,288	91,520
Path 2 1°	0,193	0,898	3,198	8,086	18,926	37,964	67,749	113,359	166,890	240,334
Path 2 10°	0,111	0,823	2,763	7,897	14,608	25,456	40,538	60,932	83,399	175,879
Path 2 100°	0,114	0,868	2,921	6,190	11,582	20,288	31,874	47,114	67,325	94,081
Path 3 1°	0,309	3,369	54,841	113,994	0,000	0,000	0,000	0,000	0,000	0,000
Path 3 10°	0,324	2,395	11,343	27,405	53,639	97,185	171,605	252,785	344,304	503,813
Path 3 100°	0,284	2,384	8,680	21,999	45,443	84,939	141,645	217,743	307,705	456,498

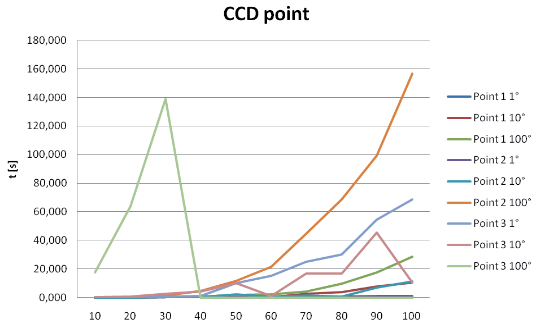

Fig. 2: *The speed of achieving points with CCD method.*

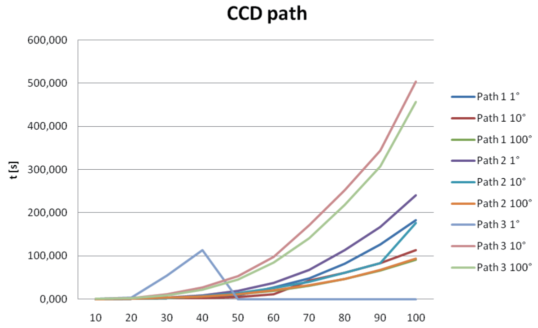

Fig. 3: *The speed of achieving a defined point with the end point of mechanism on the path with CCD.*

3.2 Incremental cyclic coordinate descent

Fig. 4 a 5 shows graphs of times for the individual computation for a method iCCD. The computation the path at point 3, step size set to 1° and the number of links 50 and more. The mechanism has not reached the desired point after n*10000 cycles. For this reason there are zero values in the table, tab. 3 and 4.

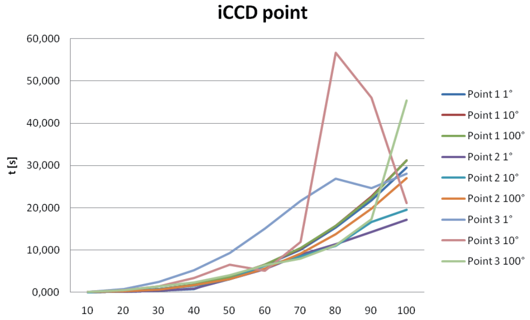

Fig. 4: *The speed of achieving points with iCCD method.*

Table 3: *Times for calculating the point with iCCD method.*

time[s]	10	20	30	40	50	60	70	80	90	100
Point 1 1°	0,035	0,073	0,309	0,776	3,782	6,417	10,135	15,312	21,706	29,543
Point 1 10°	0,026	0,212	0,713	1,835	3,719	6,471	10,364	15,658	22,627	31,258
Point 1 100°	0,038	0,209	0,735	1,823	3,664	6,504	10,419	15,661	22,406	31,169
Point 2 1°	0,056	0,148	0,324	0,937	3,097	5,533	8,904	11,435	14,279	17,150
Point 2 10°	0,022	0,184	0,690	1,511	3,133	6,230	8,534	10,950	16,620	19,466
Point 2 100°	0,043	0,195	0,651	1,647	3,168	5,657	9,120	13,781	19,830	26,949
Point 3 1°	0,141	0,755	2,449	5,170	9,254	15,010	21,562	26,845	24,706	28,009
Point 3 10°	0,118	0,539	1,344	3,364	6,526	5,140	11,910	56,669	46,003	21,148
Point 3 100°	0,101	0,505	1,430	2,319	3,998	6,405	7,941	11,140	17,332	45,435

Table 4: *Times for calculation desired point on the path with iCCD method.*

time [s]	10	20	30	40	50	60	70	80	90	100
Path 1 1°	0,129	0,493	2,256	6,056	14,873	27,046	45,013	76,148	111,710	176,468
Path 1 10°	0,062	0,435	1,717	5,044	11,843	23,064	15,347	23,578	79,357	48,142
Path 1 100°	0,073	0,493	1,999	4,964	9,861	17,077	27,102	41,341	58,715	82,051
Path 2 1°	0,179	0,864	3,012	7,829	18,570	36,683	70,122	104,240	169,993	248,164
Path 2 10°	0,141	0,783	2,613	7,610	14,005	22,367	36,287	57,265	80,107	107,370
Path 2 100°	0,099	0,874	2,862	6,206	11,528	20,013	31,161	45,986	65,892	90,376
Path 3 1°	0,310	3,319	23,524	114,173	0,000	0,000	0,000	0,000	0,000	0,000
Path 3 10°	0,317	2,267	10,997	26,712	52,412	94,981	163,662	233,079	330,214	476,499
Path 3 100°	0,276	2,372	8,566	21,666	44,685	83,221	138,506	213,658	302,851	444,241

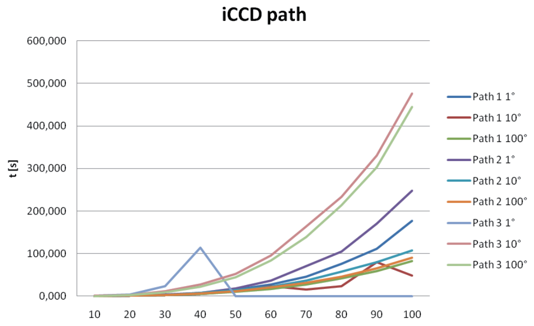

Fig. 5: *The speed of achieving a defined point with the end point of mechanism on the track with iCCD method.*

4. Shape of n-link chain

As a further criterion for the select right inverse kinematic calculation algorithm can by the shape of mechanism after achieving the desired position. This section includes a results from the calculation when the start point has same characteristic like before and the end point has characteristic of the second point from a previous part. The mechanism has shape with 100 links.

4.1 Cyclic coordinate descent

Disadvantages of CCD method is that when is setup higher value of the max step size. The calculation is quick, but the final shape of mechanism is twisted. This happens with the smaller number of links. If there is higher number of links and the large distance between the start point and the end point, the mechanism didn't reached the end point (tab.1). Like is shown on Fig. 6 for the max angle step size 90° and 180°.

For 100 links, maximal cycle number n*10000 and maximal angle step 1° reached the desired point for 8207 cycles, and for maximum step 90° and 180°, the mechanism didn't get to the desired position.

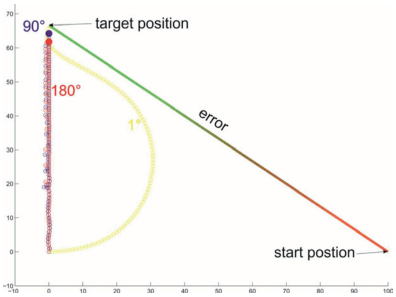

Fig. 6: *Position of serial chain with 100-link with three types of step angle with CCD method.*

4.2 Incremental CCD

With this method are all shapes about the same regardless on the max angle step size. Like is shown on Fig. 7. Every mechanism achieved the desired position for about 32 500 cycles.

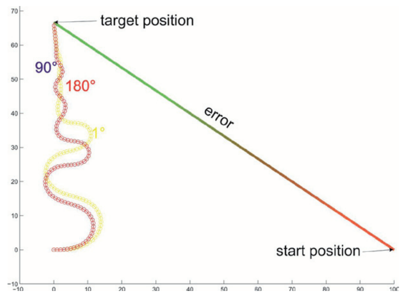

Fig. 7: *Position of serial chain with 100-link with three types of step angle with iCCD method.*

5. Conclusion

The inverse kinematics of mechanical systems is a complex issue. To a date, there are several methods for its solution. Not all are applicable to mechanisms with more degrees of a freedom because of the complex and the subsequent compilation of Jacobian and his numerical complexity. This article presents a comparison of the CCD method and its modified version incremental CCD in terms of a speed of calculation, the number of cycles, but also shape of mechanism after achieving the desired point.

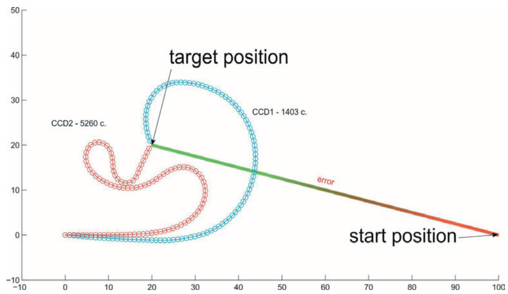

Fig. 8: *Difference of shapes between CCD and iCCD method.*

Calculation with the CCD method has reached a low number of cycles to achieve the desired position but maximal angle step size must be setup to value 1°. For higher values, there is the "twist" of mechanism and the subsequent slow convergence to the desired point. The iCCD first leads to movement of end links with successive addition of links, with which performs the movement, which is advantageous from the view point of the shape of mechanism for achieving the desired position when the end point is orientated towards the desired point. There is no way to twist mechanism, at the expense of cycles and the time.

The results show that the calculation method by the CCD is advantageous for "finding" a single point and the iCCD method is turn advantageous to move the end point of the mechanism on the pre-defined trajectory when the time is short compared to the calculation method CCD. The final confirmation of this conclusion for the whole workspace mechanism is necessary to perform additional calculations and simulations.

6. Acknowledgement

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