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Dynamic Analysis of the Two-Mass System to Imitate Rectilinear Motion of a Snake

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KEY WORDS

Snake, Locomotion, Rectilinear, Two-Mass System

ABSTRACT

A few previous decades the researchers and the designers started to copy the animal motion to the mechanisms. The principal motivations of the snaky locomotion are the environments where the traditional machines are not applicable due their dimensions or shapes and where the accessories like the wheels or the legs fail. The paper deals with the dynamic analysis of the two-mass system imitating the snake rectilinear motion on the level surface. Within the research there were made various modifications of the system and through the simulation was verified the behavior of the kinematic parameters. Based on the mentioned displacements and the speeds depending on

the time it is possible to determine the optimal parameters of examined dynamic system.

INTRODUCTION

The snakes live world wide and in variety of environment. Their way of motion and their physiological functions enable their adaptability in wide range of the landscape and variety of environment. The standard wheel robots motion is smooth and effective on the planes, but not on the changeable terrain where, in comparison, the robotic limbless snakes and robotic snakes without the wheels may do the stable motion. The robotic snakes strive to adapt the snake locomotion with the advantages of their behavior for the robotics. Their relatively small dimension makes them effective in narrow spaces. Due all these advantages the robotic snakes are perspective for extensive applications within multiple spheres. [3][4][7]

BIOLOGICAL CONSTRUCTION OF A SNAKE

The biological snakes are unusual among the other earth-born animals. The snakes are capable to do a wide range of movement from creeping through climbing to swimming. As far as all snakes are of a similar structure they exist in various sizes and relations. In contrary to the animals with the legs and capable to walk and climb, the snakes use their bodies for impacting the earth surface or against the obstacles and this way they perform the motion. It is enabled by the unusual structure of their skeleton.

THE STRUCTURE OF THE SNAKE SKELETON

This structure includes only three types of the bones- the skull, the vertebra and the ribs. The backbone may consist of 100 up to 400 vertebra. In spite of fact that any vertebra may move only from 10 to 20° in horizontal direction and from 2 up to 3° in vertical direction of the movement. However, by the combination of more vertebras the snake is capable to create a satisfactory big rotation. The rotation among the vertebras is very limited and the tissues and the muscles are preventing their dilatation.

THE SNAKE SKIN

The skin of the snake is very important during his locomotion. During the most of types of the snake movements the body of the snake is in stable contact with the surface so that the motion could be optimal. The coefficient of the snake friction is anisotropic, it means, that depending on the direction of the snake movement the moving force of the snake on the weight unit is different. It is caused by the skin surface of the snake. The skin is of high elasticity and it consists of several layers. It is covered with the scales with free rear part and the front part is partial covered with the scales positioned in front of it. The geometry of any scale and the cross disposition enable the forward snake motion with the friction of lower intensity than with the back motion. The detailed examinations at the microscopic level found that the scales are covered with small notches creating the sliding. [3]

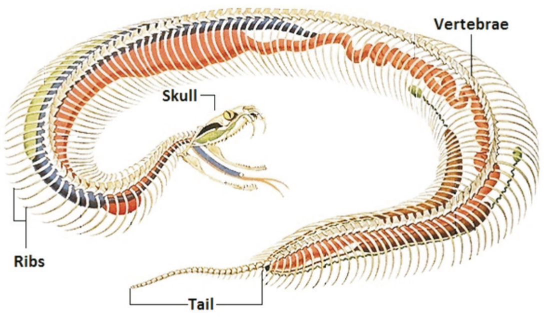

Fig. 1 *Skeleton of a snake*

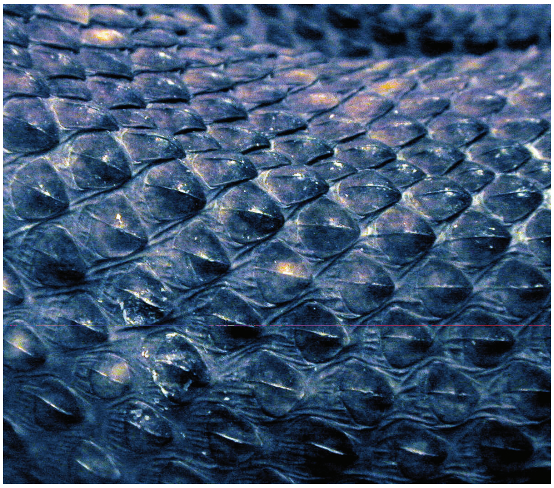

Fig. 2 *The snake skin*

THE TYPES OF THE SNAKE MOTION

h The snakes are capable of one of the most unusual way of locomotion and at the first sight it is hardly understandable, how they can move. Most of the models of the robotic snakes movement is inspired by the snake locomotion. The locomotion of the snake depends on the snake type and on the environment of its movement. Generally the snakes are moving in four ways of the following movements:

- lateral undulation,
- concertina,
- sidewinding,
- rectilinear.

RECTILINEAR MOTION

The rectilinear motion is the second basic motion of the snakes and it absolutely differs from the other ways of the motion. The rectilinear motion is specific for the snakes with the stout body disabling their side flexion. This type of motion is slower than the other ones. The abdomen is used for providing the drive through the anchoring. During the motion the abdominal scales are alternately smoothly lifting up from the surface and drawing forward and then lowering down. The parts of the abdomen skin are drawing forward and so the abdominal scales are joining in the bunch. This part of the body is then pushing down and the sides of the abdomen go down on the surface. This motion enables the snake going ahead in one line. So that to enable this mode of motion the snake fixes several points of longitudinal lower part and moves the body parts among them. The points are called as static or fixed points. The moving power of the snake is primary the frictional force and it is the force between the snake and the surface. In contrast with the cross-wave motion and with the side motion including the unilateral muscle activity changing from one side to another, the rectilinear motion includes a bilateral activity of the muscles joining the skin with the skeleton. [2][5][6]

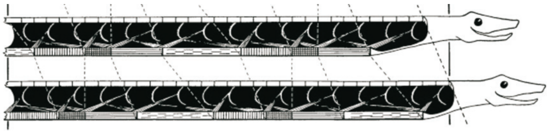

Fig. 3 *Rectilinear motion*

The Fig. 3 shows the course of the rectilinear movement of the snake. So that this movement may be examined the body of the real snake has to be transformed to the model that could be mathematically described. The next picture shows the simplified model of the snake consisting of N consecutive masses of the weight m.

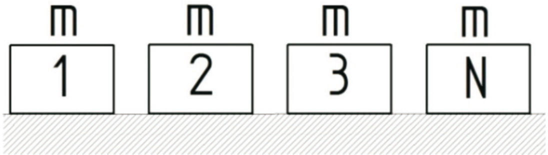

Fig. 4 *The simplified model of the snake*

The mentioned model was simplified according to the next picture where N-1 of masses was replaced with one of the weight M. This model of the snake structure was inspired by the work of professor F.L. Chernousko, where the connection between masses by using spring and dumper were not regarded. [1]

Fig. 5 *The two-mass system*

The analysis of the rectilinear motion will be realized at the mentioned model. The rectilinear motion of two moving masses will be divided in to two phases namely the slow phase where the body of the weight m will be attracted to the body of the weight M (hereinafter the body m and the body M) so that the body M stays at rest. The second phase will be the fast phase where the body m and the body M are abducted from each other. The body M is sliding forward and by repeating of the two phases this system of two masses shall carry out the forward motion.

The slow phase of the motion

This phase will be realized in two sections. During the first section the body m will be attracted to the body M while the body M stays at rest. During the second section of this phase the motion of the body m will be slowed down by the force of the same intensity as it was attracted. Fig. 6 shows the first section of the slow phase of the motion.

From the equations of this dynamic system we are obtaining the relation of the moving power F_1 =

kMg (k - friction coefficient, M - weight, g - acceleration of gravity). Maximum speed reached by the body m is expressed through the relation:

$$v_m(t) = \frac{kg(M-m)}{m} T_1 \qquad (1)$$

Where T_1 is the time, during what the attracted body m starts to be slowed down by the force of the same intensity.

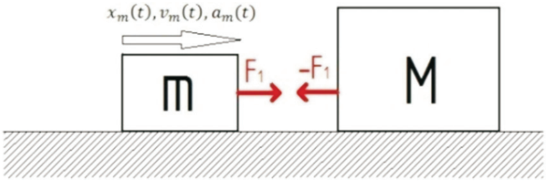

Fig. 6 *The slow phase of the motion*

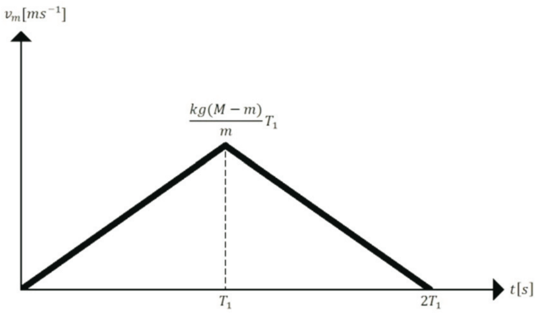

Fig. 7 *The velocity of the mass m during slow phase*

By calculation of the area capacity limited by the speed course and by the time we get the formulation of the path treaded by the body m during the slow phase. From this it is possible to express the time T_1.

$$T_1 = \sqrt{\frac{ms}{kg(M-m)}} \qquad (2)$$

Where s is maximum range of the motion between the body m and the body M.

The fast phase of the motion

During this phase occurs the forward motion due to abduction of the both bodies from each other. As slow phase the same this phase shall run in two sections. During the first section the bodies will be abducted from each other by the moving power F_2 and during the second section they will be slowed down by the force of the same intensity. Fig. 8 shows the situation of the first section of the motion.

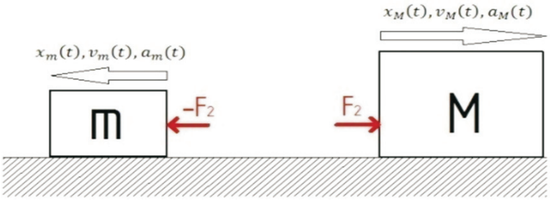

Fig. 8 *The fast phase of the motion*

During the fast phase the frictional force affecting the body M is substantially lower than the propulsive force, thus it may be ignored in solving. Maximum speed reached by the body M is expressed by the relation:

$$v_M(t) = \frac{F_2}{M} T_2 \qquad (3)$$

Where T_2 is the time during what the body M starts to be slowed down by the force F_2.
The time taking for the fast phase is very short in comparison to the slow phase and hence it may be ignored. After these analyses it was possible to determine the average speed of two moving masses. The average speed is expressed by the relation:

$$v_p(t) = \frac{1}{2(m+M)} \sqrt{kgms(M-m)} \qquad (4)$$

As the average speed of this dynamic system depends also on the weight of the moving bodies it was essential to make the analysis regarding what has to be the relation of the masses M to mass m so that the average speed could be maximal. Through the calculation of the local extreme we get the stationary point where the function of the average speed reaches its local maximum. Thus two-mass system shall reach the maximal average speed with the implication:

$$M = 3m \qquad (5)$$

After the substitution of specific figures for calculation of the body m speed during the slow phase and the body M speed during the fast phase it was found that the system need an actuating unit with the speed of motion of the tenths of the meters per second. However the commercially available actuating units capable to develop such speed are of far exceeding dimensions in comparison to the dimensions required for probable construction of an operational model. From that reason it is necessary to lower the speed of the dynamic system.

MODIFIED DYNAMICS SYSTEM

So that to lower the average speed of the dynamic system it was necessary to reconfigure it by adding the dynamic damper device between the both masses. Fig. 9 shows the modified dynamic system.

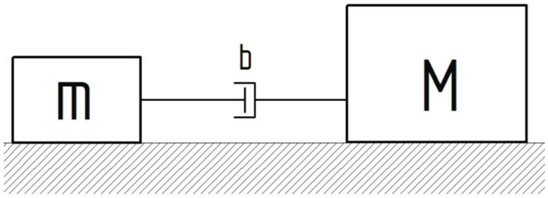

Fig. 9 *Modified dynamics system*

First it was necessary to find out if the body M stays in rest during the body m motion in course of the slow phase. Fig. 10 shows the course of the sum of the active forces affecting the body M, i.i. $F_h - F_{damper}$.

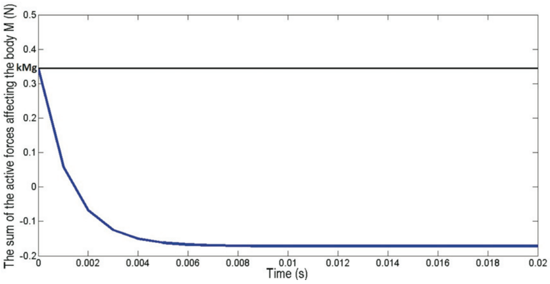

Fig. 10 *The course of the active forces affecting the body M*

Based on mentioned course we may write the relation as follows:

$$\left| F_h - F_{damper} \right| \le F_{trM} \tag{6}$$

Where F_{trM} = kMg is the frictional force affecting the body M. From this relation we may see that the frictional force is higher than the sum of the active forces, it means that the body M motion does not occur during the slow phase within the modified system.

The slow phase of the modified dynamic system motion

As during the slow phase the body M motion does not occur we may simplify this dynamic system as follows:

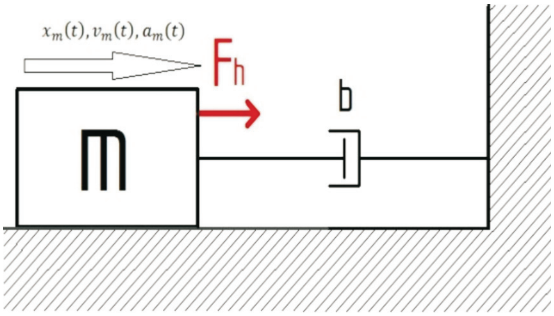

Fig. 11 *Simplify dynamic system*

The body m speed refers to:

$$v_1(t) = \frac{(M-m)kg}{b}\left(1 - e^{-\frac{b}{m}t}\right) \tag{7}$$

After finishing the affection of the moving power F_h, the body m speed is equal to:

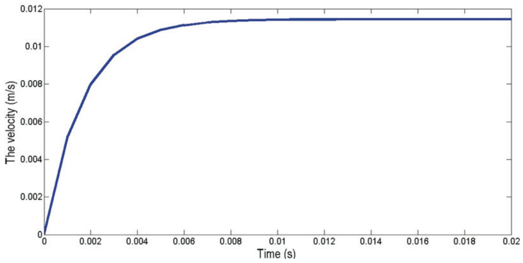

Fig. 12 *The velocity of the mass m during the slow phase*

Fig. 12 shows the body m speed course:

$$v_1(t) = -\frac{kmg}{b}\left(1 - \frac{M}{m}e^{-\frac{b}{m}t}\right) \tag{8}$$

Fig. 13 shows the course of it:

The fast phase of the modified dynamic system

During this phase occurs the difference in comparison to the previous considerations. The fast phase will consist of three sections. During the first section the both bodies will be abducted from each other by the moving power F_h.

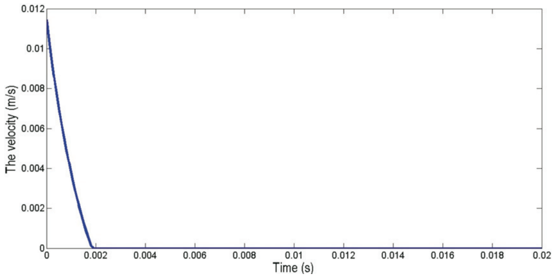

Fig. 13 *The velocity of the mass m at the end of the slow phase*

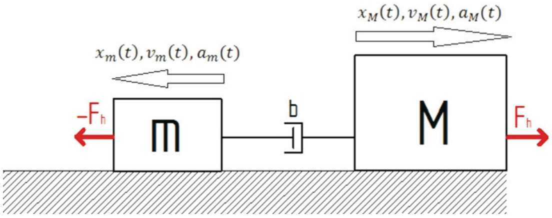

Fig. 14 *Fast phase of the modified dynamic system motion*

The relation for the body m speed will be equal to:

$$v_1(t) = \frac{b(F_{trm} - F_{trM})}{bm_c}\left[\left(\frac{M(F_{trm} - F_h)}{b(F_{trm} - F_{trM}} - \frac{m_r}{b}\right)\left(1 - e^{-\frac{b}{m_r}t}\right) + t\right] \quad (9)$$

Where $\mathrm{F_{trm}}$ and $\mathrm{F_{trM}}$ are the frictional forces and $\mathrm{m_c}$ and $\mathrm{m_r}$ relate to:

$$m_c = m + M \quad (10)$$

$$m_r = \frac{Mm}{M + m} \quad (11)$$

During the second section the body M speed is zero and only the body m is moving. The body m speed relates to:

$$v_2(t) = \frac{-F_h + F_{trm}}{b}\left\{1 + \left[\frac{bv_1^* - (F_{trm} - F_h)}{F_{trm} - F_h} e^{-\frac{b}{m}t}\right]\right\} \quad (12)$$

Where v_1^* is the initial speed during the second section reached by the body m at the end of the first section of the motion.
The third section of the motion starts with ending of the affection of the moving power $\mathrm{F_h}$ and the end of this section occurs when both bodies stop their motion. During third section of the motion the speed of the body of weight m relates to:

$$v_3(t) = \frac{F_{trm} + F_{trM}}{m_c}\left[t + \frac{MF_{trm} + bmv_2^*}{b(F_{trm} + F_{trM})} - \frac{m_r}{b} - \begin{pmatrix} \frac{MF_{rem} + bmv_2^*}{b(F_{trm} + F_{trM})} - \frac{m_r}{b} - \\ \frac{Mmv_2^*}{b(F_{trm} + F_{trM})}\frac{b}{m_r} \end{pmatrix} e^{-\frac{b}{m_r}t}\right] \quad (13)$$

Where v_2^* is the initial speed during the third section reached by the body m at the end of the second section of the motion. Next diagrams show the courses of the positions and the speeds of the body m and the body M during the fast phase of the dynamic system motion.

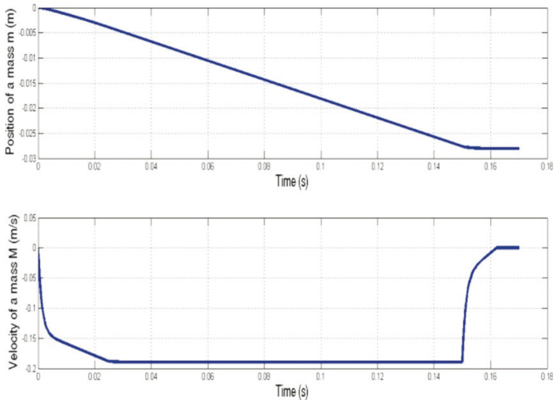

Fig. 15 *Position and velocity of the mass m during the fast phase*

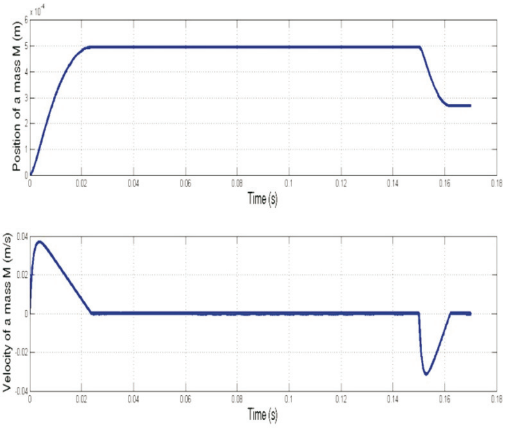

Fig. 16 *Position and velocity of the mass M during the fast phase*

After this analysis was made the examination of behavior of the dynamic system of two moving masses with the dynamic damper with added spring. This dynamic system is described by following expressions:

$$m\frac{dv_m(t)}{dt}+b[v_m(t)-v_M(t)]+k[x_m(t)-x_M(t)]=F_1-F_{trm} \quad (14)$$

$$M\frac{dv_M(t)}{dt}+b[v_M(t)-v_m(t)]+k[x_M(t)-x_m(t)]=-F_1+F_{trM} \quad (15)$$

The dynamic system was simulated in the software Matlab/Simulink and the results of the analysis are expressed by next diagrams. There are the drawings of two states of both bodies. In the first state the coefficient of the damping was constant and the spring coefficient of the elasticity varied and in consequence it was reversal.

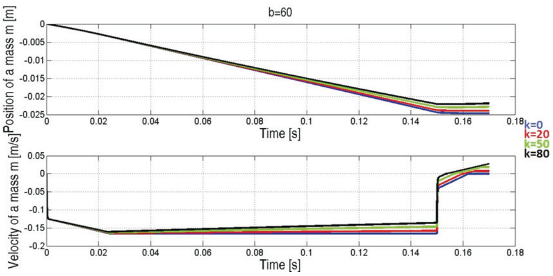

Fig. 17 *Position and velocity of the mass m during fast phase in the new modified system (b=60)*

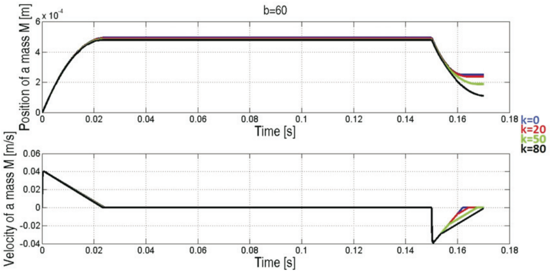

Fig. 18 *Position and velocity of the mass M during fast phase in the new modified system (b=60)*

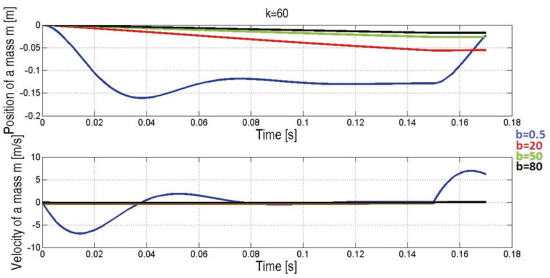

Fig. 19 *Position and velocity of the mass m during fast phase in the new modified system (k=60)*

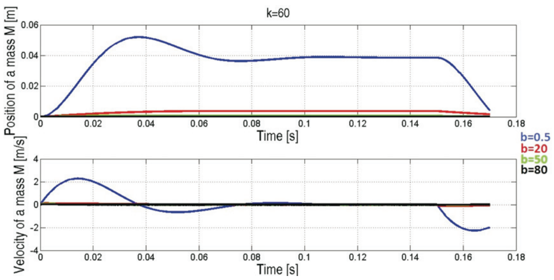

Fig. 20 *Position and velocity of the mass M during fast phase in the new modified system (k=60)*

CONCLUSION

The analysis of the rectilinear movement of the snake locomotion may be performed with simple two-mass model. Under utilization of the dynamic damper device the control of the final speed is possible by changing the coefficient of the damping. As the diagrams show the body M comes back during the last section of the fast phase and finally it means that the speed of the two-mass system substantially decreases. Up to the time 0,15 s the body M stays still in rest.

By determination of appropriate time of the fast phase course regarding the specific values (m, M, k - spring coefficient of the elasticity, b - damping coefficient) we may obtain the phase when the motion is optimum and the average speed of the dynamic system is maximal.

By adding the spring to the dynamic system it was found that by characteristic verified by us, i.i. decreasing of the speed that must be created by the actuating unit, is not markedly variable. Thus with probable design submission it is not reasonable to add the spring to the dynamic system.

ACKNOWLEDGEMENT

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REFERENCES

[1] Chernousko F. L., The optimum rectilinear motion of a two-mass system, Journal of Applied Mathematics and Mechanics, Vol. 66, No. 1, p. 1-5, 2002

[2] Dalilsafaei S., Dynamics analyze of snake robot, World Academy of Science, Engineering and Technology 29

[3] Dowling K. J., Limbless locomotion: Learning to crawl with a snake robot, Unpublished PhD Thesis. The Robotic Institute. Carnegie Mellon University, pp. 3-7, 1997
[4] Hirose S., Biologically Inspired Robots - Snake-like locomotors and manipulators, Oxford: Oxford university press, p. 2-13, 1993
[5] Hopkins J. K., Gupta S. K., Spranklin B. W., A survey of snake-inspired robot design, Bioinspiration and Biomimetics, 2009
[6] Maity A., Mandal S. K., Mazumder S., Ghosh S., Serpentine robot: An overview of current status & prospect, 14th National conference on machines and mechanisms, p. 275, 2009
[7] Shan Y., Koren Y., Design and motion planning of a mechanical snake, IEEE Transaction on systems, man, and cybernetics, vol. 23, no. 4, p. 1091-1093, 1993