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On Necessity and Possibility to Fully Exploitation of the Entire Information Inherent in Structural Analysis Data

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Structural Analysis, Condition-Monitoring, Inverse Problems and Solution, Damage-indication

ABSTRACT

The necessity of condition-monitoring and supervising of structures will be justified under aspects of reliability, durability and safety as well as with regard to economical reasons. The achievements in measurement techniques enable the evolution of efficient monitoring systems. The prerequisite will be pointed out, to conceive such systems in close co-ordination with the mathematical modelling of the structure. This is inalienable with concern to system identification as generally the control-parameters cannot be measured directly. They are to exploit on the basis of the mathematical model and the measurable structural response symptoms like displacements and strains, which inevitably requires solution of inverse problems. During service / operation many effects give rise for degradation of the structural resistance reducing the safety and the life-time as well. The results of system identification enable the determination of damage indicators, which provide information on the scale of degradation in the course of time to estimate the limit of service-life and the residual life-time.

INTRODUCTION

Supervising and condition-monitoring of technical structures must be regarded as an engineering service of ever increasing importance with regard to control durability, stability and safety as well as to estimate the life-span of structures. The nowadays complexity of new structures and strict demands on reliability and safety requires an intensive control of the structural response spectra during the entire life-time of structures. And furthermore it is unalienable to subject especially already existing structures to thorough inspections. Because during the time of existence and service/operation all and any kind of structures are more or less subjected to a permanent process of deterioration and degradation of stability, reducing the bearing capacity in the course of time and impairing the structural integrity [2], [4]. Therefore integrated strategies of health-monitoring and development of supervising systems gain high priority to assess and to guarantee the operability, reliability and safety of structures, to observe the process of deterioration and in consequence of structural degradation, in order to timely detect damages, to appraise the probability of loosing operational ability, of hazardous failure and of the (residual) life-time. This is especially very important for structures with high risk-potentials in case of failure. The use of measurement techniques is also necessary to control manufacturing and construction processes. It must be emphasised, that supervising of technical structures is to regard as an engineering service of ever increasing importance. Without any doubt the economic importance of structural condition- monitoring is evident also [5]. The processes mentioned above lead to increasing financial expenditures in measures of maintenance. Limited financial resources exclude very often the immediate replacement of structures assumed to be in bad condition. Then condition - / health-monitoring provides information whether retro-fitting might ensure safety and life-time. The aim is to avoid or at least to minimize costs and financial burden owing to:

- *operation, maintenance, repair, retrofitting or demolition and replacement,*
- *unexpected loss of operational ability,*
- *sudden unexpected collapse,*
- *consequences of hazardous damage combined with injuries, probably followed by indemnification.*

Fulfilling the previously given reasons for the necessity of structural monitoring demands the provision of a variety of data. These can be obtained by measurements under the presumption of total exploitation of the comprehensive information inherent included in the measured data.

MEASURING METHODS

The achievements in measurement techniques and the experiences in experimental solid mechanics provide useful tools and valuable support in non-destructive testing and damage analysis of structures. In this concern signal characterisation, decoding and interpretation of observed/measured phenomena have become important subjects in engineering research and practice.

The nowadays available sensor~ and measuring techniques, the techniques of recording and transmission of data combined with methods of far reaching automated data evaluation by means of powerful computer technique including comprehensive software based on proper mathematical/numerical algorithms enable installation of complex monitoring systems.

The bandwidth of the physical and technical possibilities to perform static and dynamic measurements is large [6], reaching from satellite-based GPS to fibre-optical sensors like *Bragg-gratings*. An exemplary overview is presented below, not claiming to convey a complete listing.

- *Geodetic~ / surveying techniques, satellite-based techniques;*
- *mechanical / electrical / sensor techniques;*
- *optical~, electro-optical~, optical-fibre~, optical sensor techniques, BRAGG-gratings;*
- *interferometric field techniques: holo~, shearography, ESPI, DIC, Moirè techniques, (photoelasticity);*
- *ultra-sonic~, acusto-emission~, thermo-stress~, X-ray~, tomography-techniques, etc.*

Evolving monitoring~ and supervising-systems a combination of different methods and techniques as well as a variety of sensors and instrumentations comes into question as a rule. The choice depends on the kind of structure, the environmental and surrounding conditions, and not at least on the costs of installation, operation and maintenance of the system. It is of outmost importance to evolve the system in close cooperation and coordination with the engineers responsible for the mathematical modelling of the structure. The reasons are doubtlessly evident:

- *the measuring points are to select according to the results and details of the calculations concerning the mathematical model in order to obtain powerful results of measurements,*
- *the kind and the amount of data and information, relevant for joined assessment, are to lay down with reference to the mathematical model,*
- *the mutual comparison and correlation of results enable, if necessary, an adaptation of the theoretical model.*

SYSTEM IDENTIFICATION

Comprehensive evaluation of the measured data is to regard as a task of system identification and leads in general to inverse problems, the solution of which is always based on the operator matrix ensued on the mathematical model.

The measurements furnish information on symptoms like displacements and vibration behaviour. These information is to complete by simultaneous recording environmental effects like temperature, wind-loads, moisture etc. as those effects influence the structural response during the regular measurements as well as on the long run. The symptoms and possibly additional à-priori information enable the diagnosis of the actual state of the structure and the assessment of its modifications and degradation in the course of time owing to time-depending effects like aging, fatigue, wear, creep and physical/chemical deteriorations.

The measurements first of all yield analogue "signals", either electrical, optical, acoustical, radiation and radio signals; however these signals do not come up to information necessary for relevant assessment of the actual state of the object considered. The control parameters like strength of materials, stiffness, compliance, natural values and natural frequencies, internal stress state, internal forces, which are necessary for comprehensive system identification, cannot be measured directly. They are to calculate on the basis of the recorded signals. These are to digitize and to subject statistical processes of adjustment, taking into account the defective quality of data because of noise-corrupted signals, systematic errors, outliers, lost data etc. On the basis of the thus prepared data of the symptoms and the operator matrix, i.e. the mathematical model of the structure, the related control parameters can be calculated. Inverse problems are set up immediately. The resulting systems of equations are generally improperly posed, the operator matrices are not regular, positive defined square matrices and therefore the inverse cannot set up directly. The mathematical coherence of mathematically difficult inverse problems can be found e.g. in [1], [11].

To evaluate the measured data it is absolutely necessary to dispose on a verified and validated reference model, generally the computational model of the structure. Concerning structures existing already since long time and in case of missing related documents an adequate mathematical model is to evolve, because system identification based on measured displacements cogently demands actual computational reference models.

DEFINITION OF INVERSE PROBLEMS

Any structural mechanical problem can be described by the relation

$$\mathbf{y} = \mathbf{A}(\mathbf{p}) \cdot \mathbf{x} \qquad (1)$$

with the output signals $\mathbf{y}$, the input signals $\mathbf{x}$ and the system or operator-matrix $\mathbf{A}$. In experimental mechanics let $\mathbf{y}$ denote the vector of effects: $\mathbf{y} = \{y_i\}$, $i \in [1/M]$, the elements of which can be measured, i.e. either displacements, their gradients or strain, $\mathbf{x}$ denotes the vector of causes: $\mathbf{x} = \{x_k\}$, $k \in [1/N]$, like external loading, temperature, impressed forces, constraints. The operator-matrix $\mathbf{A(p)}$, relates the input signal to the output signals and describes the structural problem depending on the vector $\mathbf{p}$, which includes all information on the parameterised boundary conditions as well as on the geometry and the material properties, or in combination of both these vectors either the vector of the stiffness- or of compliance-parameters respectively. Depending on the information sought either a forward or an inverse problem is to solve (Fig. 1).

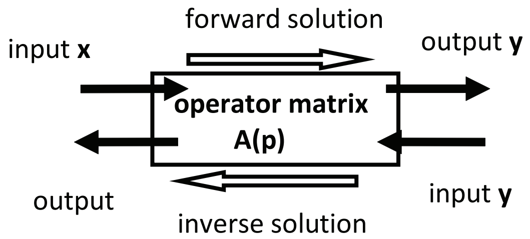

Fig. 1 *Forward/inverse solution*

Vectors and matrices are the natural data structure for discrete signals and linear operations that operate on discrete data values. Once the forward problem is encoded in matrix form, the inverse problem is set immediately.

Provided $\mathbf{A}$ and $\mathbf{x}$ to be given, then eq. (1) describes a *direct* problem, which always leads to a well-defined solution no matter, whether the mathematical model depicts the reality or not, provided a forward solver is available. But three inverse problems are posed:

- *an inverse problem of* ***cause identification****, if* $\mathbf{A}$ *is given,* $\mathbf{y}$ *has been measured and the causes* $\mathbf{x}$ *are to determine; this is an inverse problem of the 1*st *kind;*
- *an inverse problem of* ***parameter identification****, if* $\mathbf{x}$ *is given,* $\mathbf{y}$ *has been measured and parameters* $\mathbf{p}$ *included in matrix* $\mathbf{A}$ *shall be determined; this is an inverse problem of the 2*nd *kind;*
- *a* ***mixed inverse problem****, if elements of the vectors of causes* $\mathbf{x}$ *and parameters* $\mathbf{p}$ *in* $\mathbf{A}$ *are unknown and perhaps the output signals* $\mathbf{y}$ *also partly.*

All inverse problems can be seen as fitting a *"hypothesised model to measured data in order to estimate unmeasured quantities"*. Generally inverse problems are improperly posed, because the model-matrix $\mathbf{A}$ is not a square positive defined matrix and therefore cannot be inverted. Yet a *pseudoinverse* $\mathbf{A}^{-g}$ can be found always, yielding an estimate of the respectively wanted information at least.

$$\mathbf{x} = \mathbf{A}(\mathbf{p})^{-g} \cdot \mathbf{y}^{meas.} \qquad (2)$$

Numerous methods for solving inverse problems are on hand like matrix-inversion methods, iterative methods, sensitivity - matrix - based - method, artificial neural network methods, successive forward simulation, Monte-Carlo approach, genetic algorithms etc.

As a matter of fact solutions of inverse problems are not unique from the first but they are ambiguous, leading to "families" of solutions in some cases. Different solution-methods applied to the same mechanical problem and the same data can lead to completely different answers. And although the calculated results solve the initial equations describing the mechanical problem they do not render necessarily the problem to be analysed. Therefore it is of outmost importance to proof always, whether the selected solution is physically meaningful within the context of the engineering problem and to incorporate additional á-priori-information on the subject considered, furnished by the experience of the analyst. However it must be carefully checked, whether additional information probably bias the results and lead to incorrect conclusions. In this concern it must be pointed out, that the optimal solution depends on the problem to be analysed and that it is always in the responsibility of the analyst, to justify the choice of the method and to identify the physical meaning of the results.

SENSITIVITY MATRIX

The Sensitivity Matrix-based Method (SMM) [9], [10] can be categorised as an iterative method, which is especially useful in solving mixed inverse problems. The sensitivity matrix $\mathbf{S}$ is proposed relating finite changes of a vector $\mathbf{x}$ of unknown quantities to finite changes of the vector $\mathbf{y}$ of given quantities, obtained by measurements.

For estimated $\mathbf{x}^{(\mu-1)}$ and $\mathbf{x}^{(\mu)}=\mathbf{x}^{(\mu-1)} + \Delta\mathbf{x}$, $\mathbf{x}^{(\mu)}=(\mathbf{x}_k^{(\mu)})^T$, $k\in[1/N]$ the forward solver

$$\mathbf{y} = \mathbf{A} \cdot \mathbf{x} \qquad (3)$$

yields $\mathbf{y}^{(\mu-1)}$ and $\mathbf{y}^{(\mu)}$, $\mathbf{y}^{(\mu)}=(\mathbf{y}_i^{(\mu)})^T$, $i\in[1/M]$. The relation between both these vectors can be formulated as

$$\mathbf{y}^{(\mu)} = \mathbf{y}^{(\mu-1)} + \frac{\partial \mathbf{y}}{\partial \mathbf{x}}\left(\mathbf{x}^{(\mu)} - \mathbf{x}^{(\mu-1)}\right) = \mathbf{y}^{(\mu-1)} + \mathbf{S}\left(\mathbf{x}^{(\mu)} - \mathbf{x}^{(\mu-1)}\right) \qquad (4)$$

where $\mathbf{S}$ denotes the *Sensitivity Matrix*, an (MxN)-matrix, where $M\neq N$.

$$\mathbf{S} = \frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \frac{\partial (y_i)}{\partial (x_k)} \qquad (5)$$

With concern to the vector $\mathbf{y}^{(meas)}$ the elements of which are taken by measurement an are different from the calculated values of vector $\mathbf{y}^{(\mu)}$ based on estimates of $\mathbf{x}^{(\mu)}$ the error between both these vector is to minimise.

The derivative of the quadratic functional

$$J = \left\|\mathbf{y}^{(\mu)} - \mathbf{y}^{(meas)}\right\|^2 = \left\|\mathbf{y}^{(\mu-1)} + \mathbf{S}\left(\mathbf{x}^{\mu} - \mathbf{x}^{(\mu-1)}\right) - \mathbf{y}^{(meas)}\right\|^2 \qquad (6)$$

runs

$$\frac{\partial J}{\partial \mathbf{x}^{(\mu)}} = 0 = \mathbf{S}^T \cdot \mathbf{S}\left(\mathbf{x}^{(\mu)} - \mathbf{x}^{(\mu-1)}\right) + \mathbf{S}^T\left(\mathbf{y}^{(\mu-1)} - \mathbf{y}^{(meas)}\right) \qquad (7)$$

yielding J_{min} and thus an improved solution

$$\mathbf{x}^{(\mu)} = \mathbf{x}^{(\mu-1)} + (\mathbf{S}^T \cdot \mathbf{S})^{-1} \cdot \mathbf{S}^T\left(\mathbf{y}^{(meas)} - \mathbf{y}^{(\mu-1)}\right) \qquad (8)$$

and introducing finite differences eq. (4) runs

$$\Delta\mathbf{y} = \mathbf{S} \cdot \Delta\mathbf{x} \qquad (9)$$

or in matrix denotation

$$\begin{Bmatrix} \vdots \\ \Delta y_i \\ \vdots \end{Bmatrix} = \begin{bmatrix} & & \vdots & & \\ \cdots & \cdots & \dfrac{\partial y_i}{\partial x_k} & \cdots & \cdots \\ & & \vdots & & \end{bmatrix} \times \begin{Bmatrix} \vdots \\ \vdots \\ \Delta x_k \\ \vdots \\ \vdots \end{Bmatrix} \qquad (9a)$$

As generally $M\neq N$ the Sensitivity Matrix $\mathbf{S}$ is not a regular, positive defined square matrix and thus an inverse problem is on hand either overdetermined or underdetermined. In order to determine the pseudoinverse $\mathbf{S}^{-g}$, it can be fallen back upon one of the solution methods enumerated above.

$$\Delta\mathbf{x} = \mathbf{S}^{-g} \cdot \Delta\mathbf{y} \tag{10}$$

Different cases of unknown quantities in eq. (2) are to take into account. The output signals $\mathbf{y}$ (measured quantities) might be incomplete because e.g. of lost and erroneous data, the elements of the input-vector $\mathbf{x}$ might be completely or partly only unknown. For explanation the case $\mathbf{y}=(\mathbf{y}_1,\mathbf{y}_2)^T$, $\mathbf{x} = \mathbf{x}_2$ may be considered, where the known quantities are denoted by the subscript 1, the unknown by subscript 2. then eq. (10) holds

$$\begin{bmatrix} \mathbf{I} & -\mathbf{S}_{11} \\ \mathbf{0} & -\mathbf{S}_{21} \end{bmatrix} \times \begin{Bmatrix} \Delta\mathbf{y}_1 \\ \mathbf{0} \end{Bmatrix} = \begin{bmatrix} \mathbf{S}_{12} & \mathbf{0} \\ \mathbf{S}_{22} & -\mathbf{I} \end{bmatrix} \times \begin{Bmatrix} \Delta\mathbf{x}_2 \\ \Delta\mathbf{y}_2 \end{Bmatrix} \tag{11}$$

To explain the procedure the forward solver

$$\mathbf{y} = \mathbf{A}(\mathbf{p}) \cdot \mathbf{x} \tag{12}$$

will be considered. The elements of the vector $\mathbf{y}^{meas} = \{y_i\}$ have been obtained by measurement. Assuming the elements of vector $\mathbf{x} = \{x_k\}$ and of the vector $\mathbf{p} = \{p_j\}$ of the parameters are to be unknown quantities, $i \neq k \neq j$, then a mixed inverse problem is on hand and eq. (9) runs in component denotation

$$\begin{Bmatrix} \vdots \\ \Delta y_i \\ \vdots \end{Bmatrix} = \begin{bmatrix} & \vdots & & \vdots & \\ \cdots & \dfrac{\partial y_i}{\partial x_k} & \cdots & \dfrac{\partial y_i}{\partial p_j} & \cdots \\ & \vdots & & \vdots & \end{bmatrix} \times \begin{Bmatrix} \vdots \\ \Delta x_k \\ \vdots \\ \Delta p_j \\ \vdots \end{Bmatrix} \tag{12a}$$

Having ascertained the differential quotients and established the sensitivity matrix $\mathbf{S}$, for an initial estimate $(\mathbf{x}^{(0)},\mathbf{p}^{(0)})T$ a predicted vector $\mathbf{y}^{(0)}$ and the difference $\Delta\mathbf{y}^{(0)} = \mathbf{y}^{(meas)} - \mathbf{y}^{(0)}$ are to calculate. With $\Delta\mathbf{y}^{(0)}$ the solution of eq. (12a) yields the increments $\Delta\mathbf{x}^{(1)}$ and $\Delta\mathbf{p}^{(1)}$, further on improved values $\mathbf{x}^{(1)}=\mathbf{x}^{(0)}+ \Delta\mathbf{x}^{(1)}$, $\mathbf{p}^{(1)}=\mathbf{p}^{(0)}+ \Delta\mathbf{p}^{(1)}$. This procedure is to continue until $\Delta\mathbf{y}^{(\mu)}=0$ *or* $\leq \varepsilon$, a predefined tolerance.

If $M \neq N$ the system eq. (12) is improperly posed and either overdetermined or underdetermined. The method for solving the inverse problem may be chosen in the catalogue of solutions as for instance listed above. The *Sensitivity Matrix-based Method* (SMM) requires initial estimates for the unknown quantities. Proper estimation of these quantities accelerates the convergence of the calculations and is a presupposition to obtain results, which depict the reality. Additional à-priori information and rational experience of the analyst is unalienable.

Combination of SMM and FEM

The SMM has proved to be a very advantageous method in combination with the FE-method as a forward solver, especially in structural system identification [7], [8]. This will exemplarily demonstrated by applying both methods on simply supported beam (Fig. 2) without restricting the general validity.

Based on FEM the mathematical modelling of the system runs

$$\mathbf{f} = \mathbf{K}(\mathbf{H}) \cdot \mathbf{y} \tag{13}$$

where $\mathbf{K}$ denotes the total stiffness matrix with the stiffness vector $\mathbf{H}$, the vector of the nodal forces $\mathbf{f}$.

$$\mathbf{H} = \{H_\nu\}, \quad \nu \in [1/n],$$

$$\mathbf{f} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{M} \end{Bmatrix}, \quad \mathbf{F} = \{F_\nu\}, \quad \nu \in [1/(n-1)]; \quad \mathbf{M} = \{M_\nu\}, \quad \nu \in [0/n]$$

and $\mathbf{y}$ the vector of the nodal displacements

$$\mathbf{y} = \begin{Bmatrix} \mathbf{v} \\ \boldsymbol{\varphi} \end{Bmatrix}, \quad \mathbf{v} = \{v_\nu\}, \nu \in [1/n-1], \quad \overline{\boldsymbol{\varphi}} = \{\varphi_\nu \cdot l_\nu\}, \nu \in [0/n]$$

Eq. 13 is transformed now in such a formulation, that the stiffness matrix includes the displacement

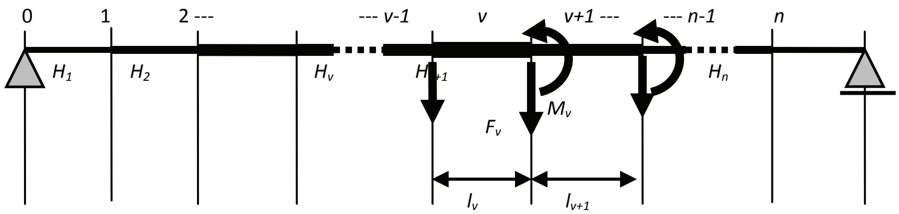

Fig. 2 *Simply supported beam*

vector $\mathbf{y}$ and the stiffness vector $\mathbf{H}$ explicitly

$$\mathbf{f} = \hat{\mathbf{K}}(\mathbf{y}) \cdot \mathbf{H} \tag{13a}$$

The matrix $\hat{\mathbf{K}}$ can be split into two matrices

$$\hat{\mathbf{K}}(\mathbf{y}) = \hat{\mathbf{K}}(\mathbf{v}) + \hat{\mathbf{K}}(\overline{\boldsymbol{\varphi}})$$

Assumed $\mathbf{y}=(\mathbf{v},\overline{\boldsymbol{\varphi}})^T$ and $\mathbf{H}$ to be unknown and collected in a vector $\mathbf{q}$ eq. 10 now reads

$$\Delta\mathbf{f} = \mathbf{S} \cdot \Delta\mathbf{q}\,, \quad with\ \Delta\mathbf{q} = (\Delta\mathbf{v}, \Delta\boldsymbol{\varphi}, \Delta\mathbf{H})^T \tag{14}$$

With the partial derivatives

$$\frac{\partial f_\nu}{\partial v_\kappa}, \kappa \in [1/n-1]; \ \frac{\partial f_\nu}{\partial \varphi_\kappa}, \kappa \in [1/n+1]; \ \frac{\partial f_\nu}{\partial H_\kappa}, \kappa \in [1/n]$$

the sensitivity matrix is set up

$$\mathbf{S} = [\mathbf{S}_V, \mathbf{S}_\Phi, \mathbf{S}_H] \tag{15}$$

with the sub-matrices depending on the unknown quantities.
It turns out, that

$$[\mathbf{S}_V, \mathbf{S}_\Phi] = \mathbf{K}(\mathbf{H}), \quad \mathbf{S}_H = \mathbf{S}_{H(v)} + \mathbf{S}_{H(\varphi)} = \hat{\mathbf{K}}(\mathbf{y}) \tag{16}$$

and eq. (13) can be formulated as

$$\mathbf{f} = [\mathbf{S}_V, \mathbf{S}_\Phi] \times \begin{Bmatrix} \mathbf{v} \\ \boldsymbol{\varphi} \end{Bmatrix} \tag{17}$$

whereas eq. (13a) runs

$$\mathbf{f} = \mathbf{S}_H \cdot \mathbf{H} \tag{18}$$

On the theoretical supposition that for given $\mathbf{f}$ all displacements as well as the stiffness parameters might be unknown these quantities are estimated for the present: $\mathbf{v}^{(0)}$, $\overline{\boldsymbol{\varphi}}^{(0)}$, $\mathbf{H}^{(0)}$.
Then an estimated vector of external loads, eq. (17), reads

$$\mathbf{f}^{(0)} = [\mathbf{S}_V^{(0)}, \mathbf{S}_\Phi^{(0)}] \times \begin{Bmatrix} \mathbf{v}^{(0)} \\ \overline{\boldsymbol{\varphi}}^{(0)} \end{Bmatrix} \tag{19}$$

the difference between given vector $\mathbf{f}$ and the estimated vector $\mathbf{f}^{(0)}$ holds

$$\Delta\mathbf{f}^{(0)} = \mathbf{f} - [\mathbf{S}_V^{(0)}, \mathbf{S}_\Phi^{(0)}] \times \begin{Bmatrix} \mathbf{v}^{(0)} \\ \overline{\boldsymbol{\varphi}}^{(0)} \end{Bmatrix} \tag{20}$$

On the other hand eq. (14) yields the difference

$$\Delta\mathbf{f}^{(0)} = \mathbf{S}^{(0)} \times \Delta\mathbf{q}^{(0)} = [\mathbf{S}_V^{(0)}, \mathbf{S}_\Phi^{(0)}, \mathbf{S}_H^{(0)}] \times \begin{Bmatrix} \Delta\mathbf{v}^{(1)} \\ \Delta\overline{\boldsymbol{\varphi}}^{(1)} \\ \Delta\mathbf{H}^{(1)} \end{Bmatrix} \tag{21}$$

Both the equations eq. 20 and eq. 21 lead to the final equation

$$\mathbf{f} - [\mathbf{S}_V^{(0)}, \mathbf{S}_\Phi^{(0)}] \times \begin{Bmatrix} \mathbf{v}^{(0)} \\ \overline{\boldsymbol{\varphi}}^{(0)} \end{Bmatrix} = [\mathbf{S}_V^{(0)}, \mathbf{S}_\Phi^{(0)}, \mathbf{S}_H^{(0)}] \times \begin{Bmatrix} \Delta\mathbf{v}^{(1)} \\ \Delta\overline{\boldsymbol{\varphi}}^{(1)} \\ \Delta\mathbf{H}^{(1)} \end{Bmatrix} \tag{22}$$

or regarding eq. (16)

$$\mathbf{f} = \mathbf{K}(\mathbf{H}^{(0)}) + \mathbf{S}^{(o)} \cdot \Delta\mathbf{q}^{(1)} \tag{22a}$$

the solution of which provides the elements of $\Delta\mathbf{q}^{(1)}$ to improve the estimated quantities in a first iteration step.

$$\mathbf{q}^{(1)} = \mathbf{q}^{(0)} + \Delta\mathbf{q}^{(1)} \tag{23}$$

The iterative procedure is to continue in μ steps until $|\Delta\mathbf{f}^{(\mu)}| \le \varepsilon$, a predefined threshold.
Let $\mathbf{f}$ be given, the components of vector $\mathbf{v}$ be measured in all nodes, $\boldsymbol{\varphi}$ and $\mathbf{H}$ are unknown. With the estimates $\boldsymbol{\varphi}^{(0)}$ and $\mathbf{H}^{(0)}$ the matrices $\mathbf{S}_V^{(0)}, \mathbf{S}_\Phi^{(0)}, \mathbf{S}_H^{(0)}$ are set up. As $\mathbf{v}$ is known the increments $\Delta\mathbf{v} = 0$. The matrix $\mathbf{S}_H$ can be split into two sub-matrices, $\mathbf{S}_H^{(0)} = \mathbf{S}_{H(v)} + \mathbf{S}_{H(\phi)}^{(0)}$ whereby the elements of matrix $\mathbf{S}_{H(v)}$ are known because of given $\mathbf{v}$. Then the final equation for calculating the increments reads

$$\mathbf{f} = [\mathbf{S}_V^{(0)}, \mathbf{S}_\Phi^{(0)}] \times \begin{Bmatrix} \mathbf{v} \\ \overline{\boldsymbol{\varphi}}^{(0)} \end{Bmatrix} + [\mathbf{S}_\Phi^{(0)}, \mathbf{S}_{H(v)} + \mathbf{S}_{H(\varphi)}^{(0)}] \times \begin{Bmatrix} \Delta\overline{\boldsymbol{\varphi}}^{(1)} \\ \Delta\mathbf{H}^{(1)} \end{Bmatrix} \tag{24}$$

where $[\mathbf{S}_\Phi^{(0)}, \mathbf{S}_{H(v)} + \mathbf{S}_{H(\phi)}^{(0)}]$ denotes the sensitivity matrix.
Let components of the vectors of displacements $\mathbf{v}$

and $\boldsymbol{\varphi}$ only partly and arbitrarily taken by measurements. These known components are marked by ${}_1\mathbf{v}$, ${}_1\boldsymbol{\varphi}$, the remaining unknown displacements by ${}_2\mathbf{v}$, ${}_2\boldsymbol{\varphi}$,. These and the stiffness parameters $\mathbf{H}$ are to determine. The vector $\mathbf{f}$ is rearranged respectively as well as the matrices $\mathbf{S}_V$ and $\mathbf{S}_\Phi$, each of which split into two matrices ${}_1\mathbf{S}_V$, ${}_2\mathbf{S}_V$, ${}_1\mathbf{S}_\Phi$, ${}_2\mathbf{S}_\Phi$. The left side index indicates whether the matrix relates to given (1) or unknown (2) displacements.

$$\mathbf{f}-\left[{}_1\mathbf{S}_V^{(0)},\ {}_1\mathbf{S}_\Phi^{(0)}\right]\times\begin{Bmatrix}{}_1\mathbf{v}\\ {}_1\overline{\boldsymbol{\varphi}}\end{Bmatrix}-\left[{}_2\mathbf{S}_V^{(0)},\ {}_2\mathbf{S}_\Phi^{(0)}\right]\times\begin{Bmatrix}{}_2\mathbf{v}^{(0)}\\ {}_2\overline{\boldsymbol{\varphi}}^{(0)}\end{Bmatrix}=$$

$$=\left[{}_2\mathbf{S}_V^{(0)},\ {}_2\mathbf{S}_\Phi^{(0)},\mathbf{S}_H^{(0)}\right]\times\begin{Bmatrix}\Delta_2\mathbf{v}^{(1)}\\ \Delta_2\overline{\boldsymbol{\varphi}}^{(1)}\\ \Delta\mathbf{H}^{(1)}\end{Bmatrix}\qquad(25)$$

It must be considered that depending on the ratio between known and unknown quantities the systems of eq.s (22), (24) and (25) respectively are in general improperly posed and therefore inverse solutions are required to obtain the pseudoinverse $\mathbf{S}^{(\mu)\text{-g}}$.

The main intention of the presented considerations is directed to the identification of the control parameters, - material properties, stiffness- or compliance parameters - on the basis of measured displacements. In this concern it must be pointed out that for different self-evident reasons a restricted number of displacements only can be provided meaningful by measurement. The remaining unknown displacements are determined in the course of the evaluation process. In addition the external loads or at least some components the load vector $\mathbf{f}$ may be unknown as well as parameterised boundary conditions included in the operator matrix of the mathematical model. On the other hand some elements of the vector $\mathbf{H}$ may be given. Thus the presented procedure enables the total exploitation of information, which are inherent included in the data taken by measurements.

DAMAGE INDICATORS AND LIFE-TIME APPRAISAL

During utilisation/operation the initial state of safety and the load-bearing-capacity will be reduced because of different reasons as already mentioned above. This leads to degradation of strength of materials and stiffness, which consequently leads towards reduction of the structural resistance $\mathbf{R}$ and also to a decrease of safety and life-span in the course of time [12], [3]. Structural failure occurs, if the external effects $\mathbf{E}$ - mainly the impressed loads - exceed the internal resistance $\mathbf{R}$ (Fig. 3).

The processes of degradation and damage are time-depending. But they can be considered as quasi-static as they do not release inertia forces. The information on the degree of damage is included in the global stiffness-matrix $\mathbf{K}$ and thus in the evolution of the stiffness H_n of the finite elements, determined in discrete time-intervals. The stiffness matrix depends on the measured displacements $\mathbf{u}$ and the degree of degradation $\mathbf{d}$. The most compact information on the damage however will be provided by the evolution of the natural values λ_m or natural frequencies ω_m of $\mathbf{K}$.

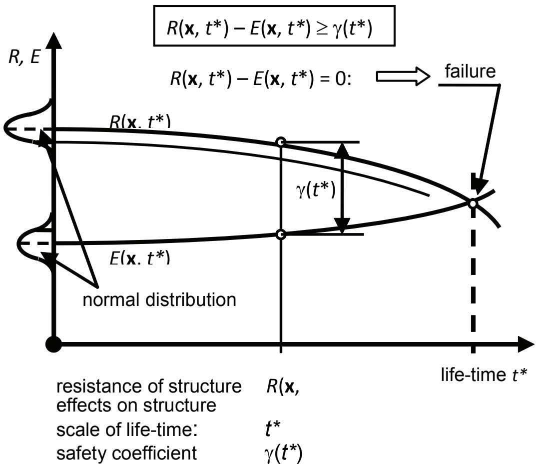

Fig. 3 *Life-time depending limit state concept*

Static analysis:

$$\left(\mathbf{K}_{(\mathbf{u},d)}-\lambda_m\mathbf{I}\right)*\boldsymbol{\Phi}_m=0,\quad m\in[1/M]$$

$\mathbf{K}$ is projected on the unity-matrix $\mathbf{I}$.

Dynamic analysis:

$$\left(\mathbf{K}_{(\mathbf{u},d)}-\omega_m^2\mathbf{M}\right)*\boldsymbol{\Phi}_m=0,\ m\in[1/M]$$

$\mathbf{K}$ is projected on the mass-matrix $\mathbf{M}$.

The natural vectors $\boldsymbol{\Phi}_m$ convey an insight of the spatial distribution of damage. In the process of degradation the natural values and natural frequencies are reduced; if finally one of them approaches 0, then *det* $\mathbf{K}=0$ and the integrity of the structure is violated and failure occurs.

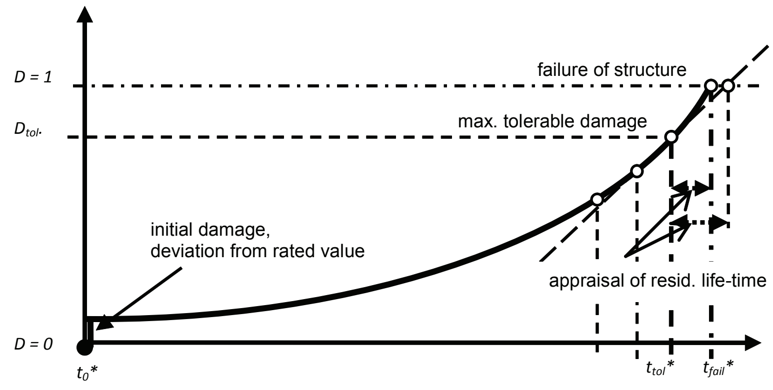

Fig. 4 *Principle course of damage indicators*

As damage indicator the definition

$$D = 1 - \frac{parameter\ in\ the\ damaged\ \ state}{parameter\ in\ the\ initial\ \ state}$$

has proved to be advantageous [13]. Any parameter, which includes information on damages, can be taken into consideration like for instance the natural values or the stiffness parameters of the finite elements.

$$D_m(t) = 1 - \frac{\lambda_m(t)}{\lambda_m(t_0)}, \quad D_n(t) = 1 - \frac{H_n(t)}{H_n(t_0)} \qquad (26)$$

The maximum value of D yields the decisive information on the state of damage.

$D = 0$: *undamaged state,*
$0 \le D \le 1$: *partially damaged*
$D = 1$: *failure*

The evolution of the indicator enables estimation of the life-time and the residual life-time respectively (Fig. 4). If D exceeds an as tolerable accepted limit no matter how this limit might be defined the end of the service life is indicated.
By a simple theoretical example the procedure of damage indication will be demonstrated assuming the element no. 4 of the beam (Fig. 5) to be subjected to corrosion. During the course of time the displacements $\mathbf{u}$ in the nodal points might have been measured. Fig. 5 shows for instance the deflection curves related to the progressive degrada-

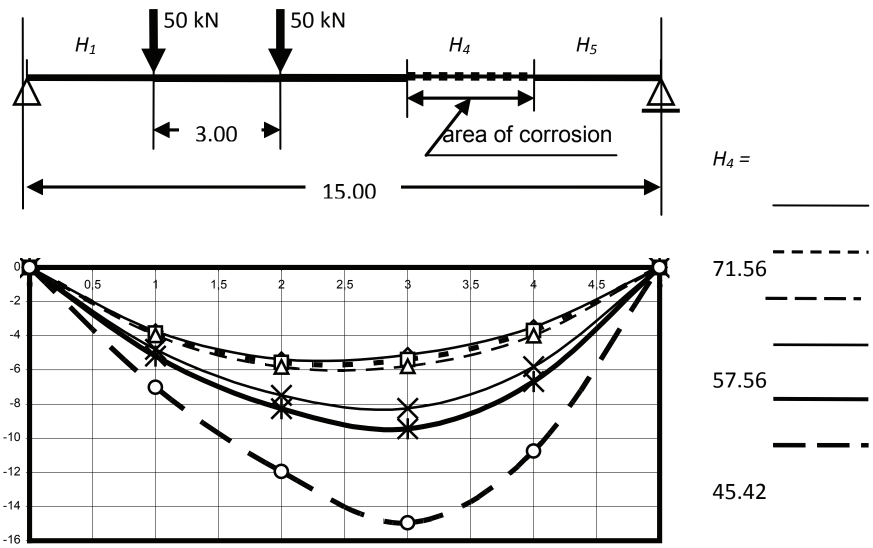

Fig. 5 *Deflection curves depending on progressive corrosion*

tion of the beam caused by corrosion in element no. 4.

The evaluation process of the measured data of displacements as described above yields the stiffness parameter H_4 and following the total stiffness matrix $\mathbf{K}_{(u,d)}$, further on the related natural values λ_m and the damage indicators D as functions of life-time t^*. The results are presented in Fig. 6.

CONCLUSION

It turns out the methods of experimental mechanics combined with proper mathematical algorithms for the evaluation of the metered data to be quite useful in non-destructive testing, health-monitoring and damage assessment of aging structures. Certainly in practice additional information and measures are necessary as for instance controlling the structure by appearance, controlling the boundary conditions and the state of supports, taking into account changes and effects of dead- and payload, environmental conditions like temperature and moisture. The goal of solving inverse problems is to determine from measured data the values of unmeasured and nonmeasurable deformations and further on the characteristic structural parameters in order to get reliable information on the respective actual state of structures. In most cases the measured data are defective, whereby it must be differentiated between accidental, systematic and proportional errors. Erroneous data affect the solutions essentially; therefore statistical processing of data is necessary to minimise the defects. But depending on the kind and the source of errors this is not possible always or at least not completely.

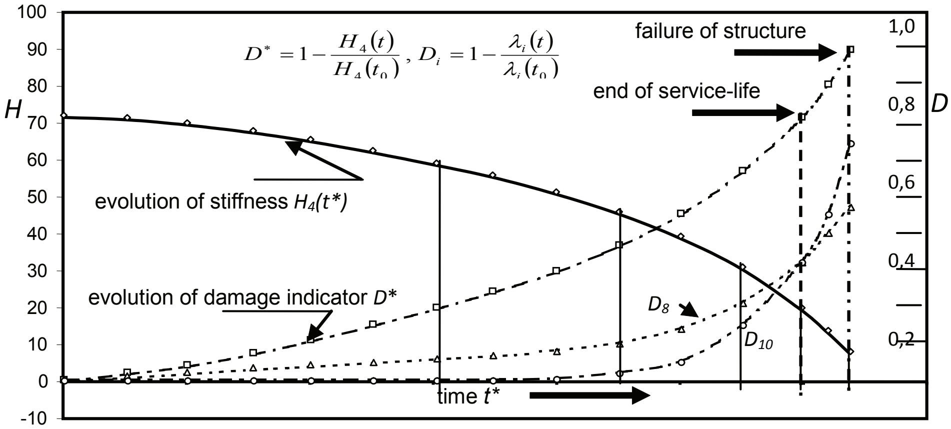

Fig. 6 *Evolution of the stiffness degradation and damage indicators*

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