

Analysis of Quality of Relay Control Systems with Correction

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BIOGRAPHICAL NOTES

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ABSTRACT

Modeling of relay control systems using Simulink toolbox of the Matlab environment is presented in the paper. The research was performed on systems with static and astatic plants with various parameters, which contained two-position on-off relays without and with correction. Simulation results show positive impact of the correction to the quality of relay control system.

GENERAL APPROACH TO RELAY CONTROL

Due to simple structure, relay control systems are widely used in both industrial and household devices. Process parameters maintained by relay control are:

- *temperature in furnaces, refrigerators, dryers, washing machines,*
- *level of liquids and loose materials stored in containers,*
- *pressure of air and water in supply installations,*
- *voltage and rotations of direct-current machines.*

Simplified structure of the relay control system, with controller in the form of two-position on off relay, is shown in Fig. 1.

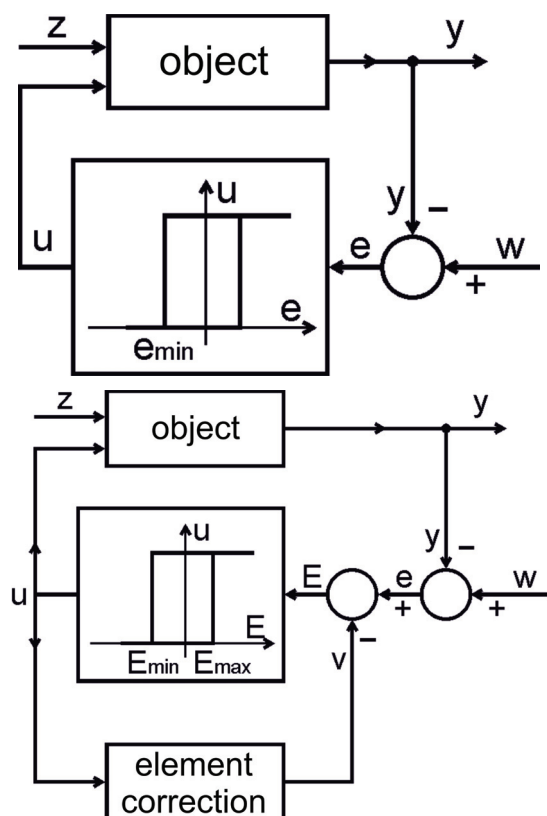


Fig. 1 Simplified block diagram of relay control system; a) relay without correction; b) relay with correction

Relay control is described by the following features:

- **controlled signal $y(t)$ oscillates with specific amplitude $\pm\Delta y$ and period T_{osc} in the vicinity of the setpoint $w(t)$,**

- **the course of the controlled signal $y(t)$ in the range of, $w-\Delta y \div w+\Delta y$, realized by turning the energy flow to the plant on and off, is the same as the time response of the plant,**

- **oscillation amplitude of the controlled signal Δy and frequency $1/T_{osc}$ of turnings the energy flow to the object on and off depend on the properties of the object and relay.**

To increase quality of control in systems with relay controllers, internal negative feedback (Fig. 1b) with 1st or 2nd-order components is used.

This creates an internal relay control system, where the setpoint is the error signal e from the main control system. In such system, oscillations of the u sig-

nal emerge with frequency dependent of the relay hysteresis loop width and dynamic properties of negative feedback component.

OBJECT OF RESEARCH

Due to their properties, dynamic control plants can be classified as static or astatic. With accuracy sufficient for industrial applications, these properties can be approximated by the transfer function as follows:

- **static plants**

$$G(e) = \frac{ke^{-T_0s}}{T_s + 1} \quad (1)$$

- **astatic plants**

$$G(e) = \frac{Ke^{-T_0s}}{T_s + 1} \quad (2)$$

where:

k – gain,

T – time constant (determined from substitute characteristic),

T_0 – time delay,

$K = \tan \alpha$ (α – angle between horizontal axis of time and response of the plant).

To show efficiency of relay control, the research on systems without correction (Fig. 1a) and with correction (Fig. 1b) for static and astatic plants with different values of parameters was performed.

The research assumed wide range of variations of controller (two-position relay) and plant parameters. Therefore:

- **for relay**

* U_{min}, U_{max} -0,1; 0,2; -1,1; -0,2; 1; -1,2; -2,2

* e lub E (for relay with correction) $\pm 0,05s; \pm 0,1s$

- **for static plant $k=2; T_0=0 \div 5s; T=1 \div 5s$**

- **for astatic plant $T_0=0 \div 5s; K=0,2 \div 2$**

The research was performed using Simulink toolbox of the Matlab environment. From the large number of obtained time-responses of the controlled signal, only several results will be shown in the paper.

RELAY CONTROL WITHOUT CORRECTION

Figure 2 shows block diagrams of relay control system for static plant with parameters $k=2; T_0=0s; T_0=0,8s; T=3s$ (Fig. 2a, Fig. 2b) and for astatic plant with parameters $K=0,5; T_0=0,8s$; (Fig. 2c).

Time-responses $y(t)$ to step input $w(t)$ are shown in Fig. 3 to 6. In the case of the system from Fig. 2a,

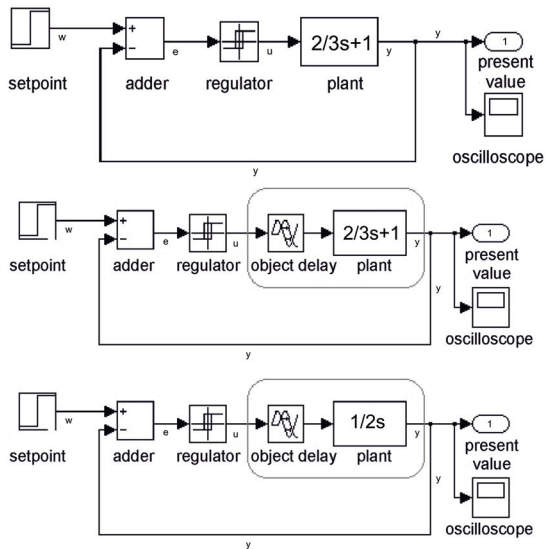


Fig. 2 Block diagram of relay control system

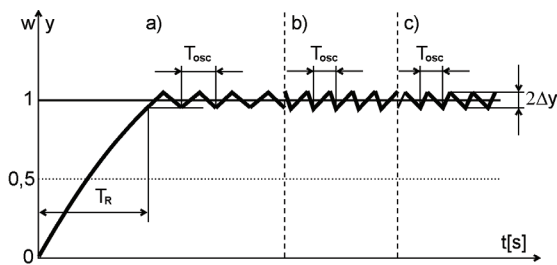


Fig. 3 Step response of the control system for static plant $T_o=0s$

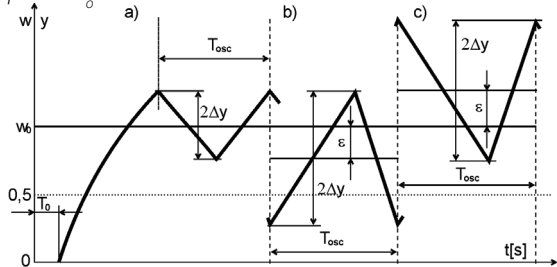


Fig. 4 Step response of the control system for static plant $T_o=0,8s$

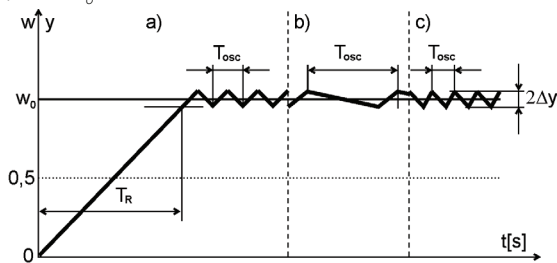


Fig. 5 Step response of the control system for astatic plant $T_o=0s$

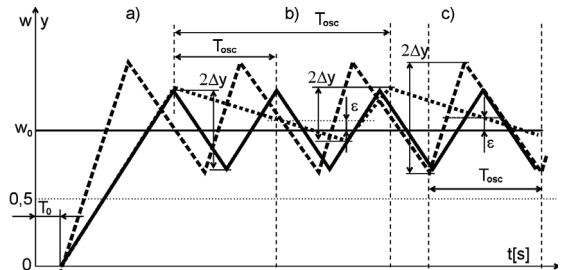


Fig. 6 Step response of the control system for astatic plant $T_o=0,8s$

parameters of the relay output signal were assumed as follows:

a) $U_{min}=0, U_{max}=1$

b) $U_{min}=-1, U_{max}=1$

c) $U_{min}=0, U_{max}=2$

the hysteresis was equal $\Delta e=\pm 0,05s$. In the case of the system from Fig. 2b, the output of the relay was assumed as follows:

a) $U_{min}=-1, U_{max}=1$

b) $U_{min}=-0,2, U_{max}=1$

c) $U_{min}=-1, U_{max}=2$

the hysteresis was equal $\Delta e=\pm 0,05s$.

RELAY CONTROL WITH PD CORRECTION

The simplest type of correction in relay control systems is realized by putting 1st order component into the negative feedback path. Block diagram of such correction is shown in the Fig. 7.

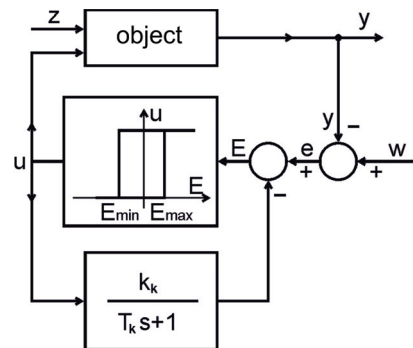


Fig. 7 Block diagram of relay control system with PD correction

Resultant structure consists of controller which transfer function can be approximately written as the inverse of the corrective element transfer function:

$$G(e) = \frac{T_k s + 1}{k_k} = k_r (1 + T_d s) \quad (3)$$

Obtained transfer function of the controller corresponds to the properties of the PD controller. Block diagrams of the control systems with PD correction are shown in Fig. 8.

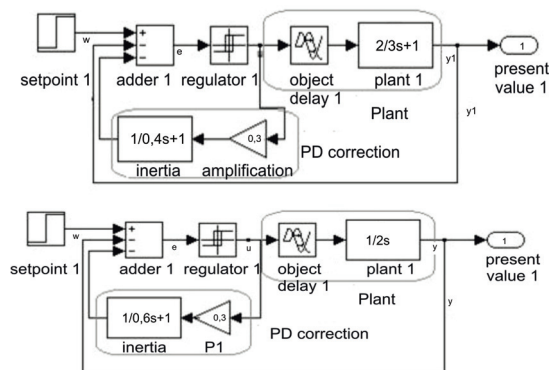


Fig. 8 Block diagram of the control system with PD correction, a) static plant with delay, b) astatic plant with delay

Simulations were performed for $k_k=0,2\div 1$ and $T_k=0,1\div 5$. Time-responses $y(t)$ of the system to step input $w(t)$ are shown in Fig. 9 and Fig. 10. The output of the relay for the system from Fig. 8a was set to $U_{min}=0; U_{max}=1$, for the system from Fig. 8b: $U_{min}=-1; U_{max}=1$, hysteresis in both cases was equal $\Delta e=\pm 0,05s$.

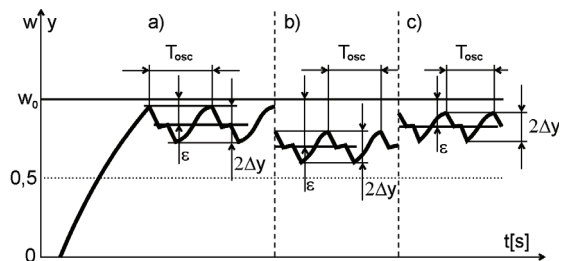


Fig. 9 Step responses of relay control system from Fig. 8a with PD correction, $T_o=0,8s$

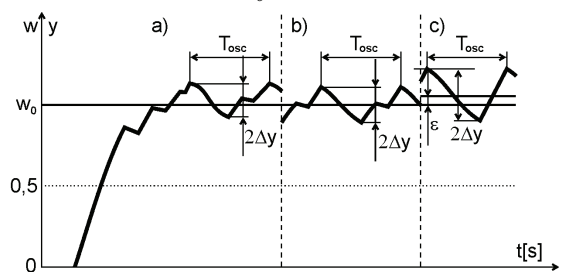


Fig. 10 Step responses of relay control system from Fig. 8b with PD correction, $T_o=0,8s$

RELAY CONTROL WITH PID CORRECTION

More sophisticated type of correction in relay control systems is constituted by two 1st order elements put into the relay negative feedback path. Block diagram of such correction is shown in Fig. 11.

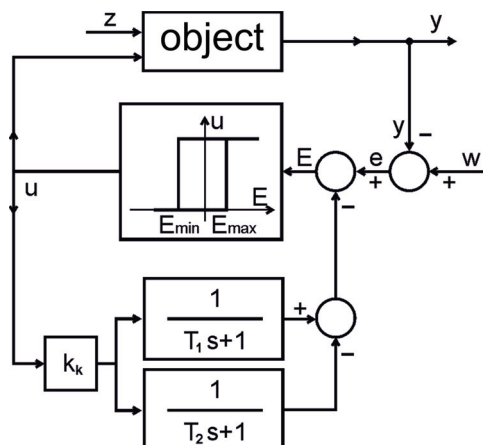


Fig. 11 Block diagram of relay control system with PID correction

With assumption that $T_2 > T_1$, resultant structure consists of controller which transfer function can be approximately written as the inverse of the corrective element transfer function:

$$G_r = \frac{1}{G_k(s)} = k_r \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (4)$$

If we denote $m = T_2 / T_1$, we get:

$$k_r = \frac{1}{k_k} \frac{m+1}{m-1} \quad \text{gain,}$$

$$T_i = \frac{T_2(m+1)}{m} \quad \text{integral time,}$$

$$T_d = \frac{T_2}{m+1} \quad \text{derivative time.}$$

Obtained transfer function corresponds to the properties of the PID controller. Block diagram of the control system with PID correction is shown in Fig. 12. Simulations were performed for $k_k=0,2\div 1$; $T_1=0,1\div 0,5s$; $T_2=0,4\div 1s$. Time responses $y(t)$ of the system to step input $w(t)$ are shown in Fig. 13 and Fig. 14. The output of the relay for the system from Fig. 12a was set to $U_{min}=0; U_{max}=1$, for the system from Fig. 12b: $U_{min}=-1; U_{max}=1$, hysteresis in both cases was equal $\Delta e=\pm 0,05s$.

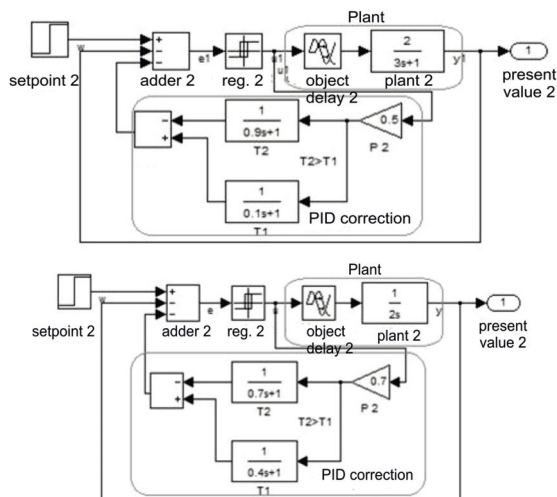


Fig. 12 Block diagram of relay control system with PID correction, a) static plant with delay, b) astatic plant with delay

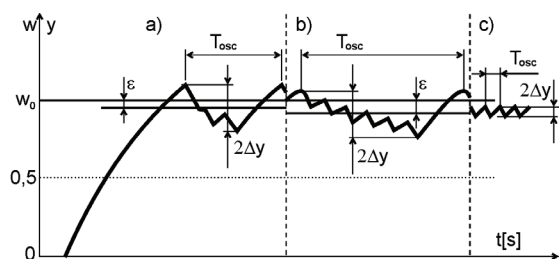


Fig. 13 Step response of relay control system from Fig. 12a with PID correction, $T_o=0.8s$

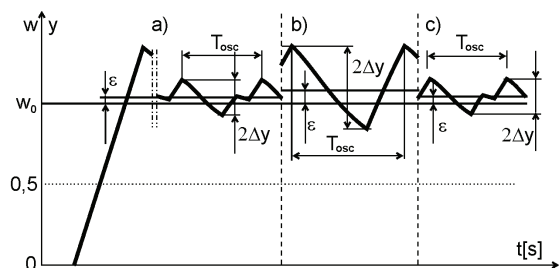


Fig. 14 Step response of relay control system from Fig. 12b with PID correction, $T_o=0.8s$

SUMMARY

The course of the controlled signal in relay control systems is characterized by average oscillation amplitude (difference between average value and maximum or minimum value) and oscillation period which is the sum of rise time and fall time of the controlled signal. As the research showed, these parameters depend on relay parameters (hysteresis,

output value) and plant parameters (gain, time constant, integral time, time delay). General conclusions are as follows:

- time constant T of static plant and integral time $T=1/K$ of astatic plant determine oscillation period of the controlled signal,
- hysteresis of the relay determines oscillation amplitude of the controlled signal,
- time delay of the plant causes increase of oscillation amplitude and oscillation period of the controlled signal,
- symmetry or the lack of symmetry of the relay output determines maximum and minimum value of the controlled signal (variations from average value).

As demonstrated, the most important problem in relay control is the time delay of the plant. In such case it is necessary to apply dynamic PD or PID correction. Although corrective elements contribute to more sophisticated structure of the system, they provide significant increase of quality of control (lower amplitude and period of controlled signal). It is worth to notice that PD correctors used in systems with static plants do not eliminate static error.

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