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Active Vibration Control of Aluminium Flexible Structure Using Optimal Controller

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BIOGRAPHICAL NOTES

Mouleeswaran Senthil kumar. Is currently working as Senior Lecturer in the Department of Mechanical Engineering at PSG College of Technology, Coimbatore, India. He obtained his B.E. Degree in Mechanical Engineering (1994), M.E. in Engineering Design (1996) and Ph.D. in Active Suspension System (2008). He has about 12 years of teaching experience and 1 year of industrial experience. He has authored 19 journal papers and about 40 conference papers. He has received ISTE award for Best Research in the field of Machine design and Vibration by Young Teacher in 2006. He has also received AICTE Career Award for Particle Damping in 2009. He has visited countries such as Portugal, United Kingdom and France for joint research works. He has completed two sponsored projects on Composite Leaf Springs and Propeller Shafts for Automotive Applications and Active Suspension System for Light Passenger Vehicles. Currently he has been the Principal Investigator of two projects in the areas of Vibration Control using Smart Structures. He has been actively involved in many consultancy works in the areas of design and vibration control. His fields of interest include vibration control, composites, smart structures, etc.

KEY WORDS

Flexible structure, active vibration control, optimal controller

ABSTRACT

Vibrations of aluminium flexible structure are actively controlled by using piezoelectric actuators. In active vibration control the control device has varying properties for controlling vibrations. In this active control system, the vibrations are sensed by the sensors which are placed at free end of the beam and these sensed signals are given as input to the control system. In turn, out put in the form of voltage is given to the piezoelectric actuator to control the vibrations by an optimal controller. Thus vibrations are controlled actively. In this paper, an optimal controller called Linear Quadratic Regulator has been designed involving minimization of the total energy of the beam along the length of the beam to control the vibrations of aluminium flexible beam. It is concluded that the vibrations due to step excitation are effectively controlled by optimal controller by varying weighting matrices.

NOMENCLATURE

F External force acting in [N]
y Deflection in [m]

M	Bending moment of the beam in [Nm]
V	Shear force of the beam in [N]
m	Mass of the beam in [kg]
a	Acceleration of the beam in [m/s^2]
ρ	Mass density in [kg/m^3]
A	Area of cross section of the beam in [m^2]
F_y	Forces along y-axis in [N]
E	Young's modulus in [N/m^2]
I	Moment of inertia in [m^4]
ε_i	Strain along 'i' th direction
d_{ij}	Piezoelectric charge constant in [m/V]
t_p	Thickness of piezoelectric patch in [m]
t_b	Thickness of the beam in [m]
L	Length of the beam in [m]
w_b	Width of beam in [m]
w_p	Width of piezoelectric patch in [m]
r_1, r_2	Locations of the patch from the fixed end
U	Voltage input to the piezoelectric patch in [V]
σ	Stress in the piezoelectric patch in [N/m^2]
q_i	Time dependent solution of the deflection for the 'i' th mode
Φ_i	Mode shape for the 'i' th mode
δ_{ij}	Kronecker delta function
r_w	Length from the fixed end at which load act ing in [m]
ζ_i	Damping ratio for the 'i' th mode
λ	Roots of the mode shape
ω	Natural frequency in [rad/sec]
δ	Logarithmic decrement
J	Cost function

Matrices

[K]	Controller gain matrix
[R]	Controller weighting matrix
[Q]	State weighing matrix

Subscripts

b	Indicating Beam
p	Indicating piezoelectric patch
i	Indicating 'i' th mode

INTRODUCTION

Flexible structure vibrations are controlled actively by using piezoelectric actuators. In this active control system, the vibrations are sensed by the sensors which are placed at free end of the beam. These sensed signals are given as input to the control system and the out put of the control system is in the form of voltage signal which is given to the piezoelectric actuator to damp the vibrations of the structure. Linear Quadratic Regulator (LQR) is used as an optimal controller to damp the vibrations throughout the length of the structure actively. When force is given as input to the beam, then the beam undergoes unsteady vibrations. These disturbances can be damped by designing the Optimal LQR which can actively control the vibrations of the beam. The LQR design includes optimal control law, which involves minimization of the total energy along the length of the beam. A code is developed using Matlab 6.1 for finding the controller gain matrix. In active vibration control structural response is controlled by adding controlling elements to structure like piezoelectric actuators. For active control of vibrations control devices properties are varying according to the changes in state.

Young Kyu Kang, Hyun Chul Park, Jaehawan Kim and Seung-Bok Choi [1] investigated the interaction between active and passive vibration control characteristics by optimal control theory using piezoelectric actuators. X.Q. Peng, K.Y.Lam and G.R. Liu [2] proposed finite element third order theory for active position control of composite beams. A special type of collocated feedback controller for smart structure was proposed by H.R.Pota, S.O.Reza Moheimani and Matthew Smith [3]. They presented the way in which resonant amplitudes of vibrations are controlled by using piezoelectric actuators and sensors. Reza Moheimani, Hemanshu R.Pota and Ian R Petersen [4] presented the application of piezoelectric materials in active control of unwanted vibrations in flexible structure using spatial control. Dunant Halim and S.O.Reza Moheimani [5] designed a feedback controller to suppress vibrations of a flexible beam. The suppression is based on the spatial H2 norm. Y.Y. Lee and J. Yao [6] proposed the way in which piezoelectric sensors and actuators are used for structural vibrations suppression. The independent modal space control approach is employed by them for the controller design. S.O Reza Moheimani [7] proposed the recent innovations in vibration damping and control using shunted piezoelectric transducers. A. Baz and S. Poh [8] proposed the utilization of piezoelectric actuators in controlling the structural vibrations of flexible beams. In this work vibrations of flexible structures caused by disturbing forces are actively controlled by using piezoelectric actuators. Optimal LQR is designed for controlling the vibrations of the structure. Controller is designed based on the optimal control law which is going to minimize the cost function.

ACTIVE VIBRATION CONTROL

In passive vibration control, the control devices have constant properties. But in active vibration control, the control devices are having varying properties which can control the vibrations actively. Here, the vibrations are sensed by sensors placed at the free end of the beam. The deflections are given as input to the controller. The controller calculates the controlling voltages required for the piezoelectric patch to control vibrations. The calculated voltages are given as inputs to the piezoelectric patches. Fig.1 shows a flexible aluminium cantilever beam with sensor, controller and piezoelectric actuators. Voltage required to the piezoelectric (PZT) actuator for vibration control, is connected by feedback loop as shown in Fig. 2.

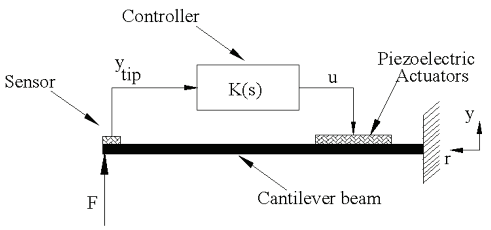

Fig. 1 *Cantilever beam with Active Control System*

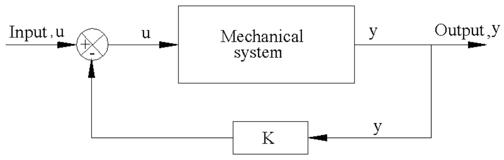

Fig. 2 *Linear quadratic regulator*

ASSUMPTIONS

Following are the idealizations made for the active vibration control of flexible structures.

- *Beam is subjected to pure bending and the material is isotropic and homogeneous.*
- *The material obeys Hooke's law and the beam is initially straight with a cross section that is constant throughout the beam length and cross sections of the beam remain plane during bending.*
- *Damping effect due to bonding layer is not considered.*

Table 1 shows the parameters of the beam and PZT actuator considered.

Aluminum Beam parameters		**Piezoelectric material parameters**	
Beam material	***Aluminum***	***Piezoelectric material***	***PZT***
Young's Modulus	6,7x1010 [N/m^2]	Young's Modulus	6,7x1010 [N/m^2]
Beam length	0,775 [m]	Patch length	0,07 [m]
Beam width w_b	0,05 [m]	Patch width w_b	0,025 [m]
Beam thickness t_b	0,00589 [m]	Patch thickness t_b	1x10^{-3} [m]
Mass density	2700 [kg.m^{-3}]	Charge Constant, d31	-210x10^{-12} [m.v^{-1}]
Moment of Inertia, $I=w_b t_b^3/12$	8,51401e^{-10} [m^4]	Voltage Constant, g31	-11,5x10^{-3} [m.N^{-1}]
Area of cross section of beam, $A= w_b\, t_b$	0,0002945 [m^2]	Coupling Coefficient	0,34

Tab.1 *Parameters of Aluminum beam and PZT patch*

BERNOULLI-EULER BEAM EQUATION

By considering f(r) external force acting on the Bernoulli-Euler beam equation of beam is given by,

$$EI\frac{\partial^4 y}{\partial r^4} + (\rho A)\frac{\partial^2 y}{\partial t^2} = f(r) \qquad (1)$$

Active vibration control of flexible structures is done by using piezoelectric actuators. When an electric voltage is applied to piezoelectric patch, it produces strain both in longitudinal and transverse directions. Piezoelectric strain constant is the ratio of developed free strain to the applied electric field. Of particular importance are the strain constants d_{33}, d_{31} and d_{32}. The subscript d_{ij} implies that the voltage is applied or charge is collected in the 'i' direction for displacement or force in the j direction. While the transducer is in the actuator mode resultant transverse strains is given by,

$$\varepsilon^{l} = \left(\frac{d_{31}}{tp}\right)U(r,t). \qquad (2)$$

Stress induced in the piezoelectric patch along the length.

$$\sigma = E_p \varepsilon = \left(\frac{E_p d_{31}}{t_p} \right) U(r,t) \quad (3)$$

Bending moment due to the stress along length of the piezoelectric patch is given by,

$$M_p = \int_{\frac{t_b}{2}}^{\left(\frac{t_b}{2}+t_p\right)} \left(\frac{E_p d_{31}}{t_p} \right) U(r,t) w_p y \ x \ dy \rightarrow$$

$$\rightarrow M_p = C_p U(r,t), \text{ where } C_p = \frac{1}{2} E_p d_{31} w_p (t_b + t_p) \quad (4)$$

ASSUMED MODES APPROACH

When the piezoelectric actuator is patched on the beam, then the equation of beam is given by,

$$EI \frac{\partial^4 y(r,t)}{\partial r^4} + (\rho A) \frac{\partial^2 y(r,t)}{\partial t^2} = f(r,t) + C_p \frac{\partial^2 U(r,t)}{\partial r^2} \quad (5)$$

The main idea of the assumed modes approach is to expand the function y(r, t) as an infinite series in the following form

$$y(r,t) = \sum_{i=1}^{\infty} q_i(t) \phi_i(r) \quad (6)$$

where Φ_i(t) are the eigen functions or mode shapes satisfying ordinary differential equations and boundary conditions of the cantilever beam. Substituting y(r, t) into the equation (5) then multiplying the resulting equation by Φ_j(r) and integrating over [0,L] we have,

$$EI \int_0^L \sum_{i=1}^{\infty} \phi_i''''(r) q_i(t) \phi_j(r) dr +$$

$$+ \rho A \int_0^L \sum_{i=1}^{\infty} \phi_i(r) q_i''(t) \phi_j(r) dr = \quad (7)$$

$$= \int_0^L f(r,t) \phi_j(r) dr + C_p \int_0^L \frac{\partial^2 U(r,t)}{\partial r^2} \phi_j(r) dr$$

From these orthogonal conditions and by using the following equation beam equation derived.

$$\phi_i''''(r) = \lambda^4 \phi_i(r) \quad (8)$$

Bernoulli-Euler equation is modified to the following form,

$$\rho A L^3 \left(q_i''(t) + \omega^2 q_i(t) \right) = \phi_i(r_w) F(t) +$$

$$+ C_p \left[\phi_i'(r_1) - \phi_i'(r_2) \right] U(t) \quad (9)$$

Equation (9) is the equation of beam is a function of time variable. When the beam vibrations are not controlled then the controller term will not be there in the equilibrium equation. When the modal damping of the beam considered then the equation (9) transformed to the following equation form.

$$\rho A L^3 \left(q_i''(t) + 2\xi\omega \ q_i' \ \ \omega^2 q_i(t) \right) =$$

$$= \phi_i(r_w) F(t) + C_p \left[\phi_i'(r_1) - \phi_i'(r_2) \right] U(t) \quad (10)$$

MODE SHAPE

Beam will be having infinite number of natural frequencies. But only first four natural frequencies the beams are considered and actively controlled. Boundary conditions of cantilever beam are given by,

$$y(0,t), \ EI \frac{\partial y(r,t)}{\partial r} = 0, \ EI \frac{\partial y^2(L,t)}{\partial r^2} = 0$$

$$\text{and } EI \frac{\partial y^3(L,t)}{\partial r^3} = 0 \quad (11)$$

After substituting the boundary conditions from equation (11) into the solution of differential equation (8), then mode shapes are given by following equation

$$\phi_i(r) = L \left[\begin{array}{l} \cosh \lambda_i r - \cos \lambda_i r - \left(\frac{\cosh \lambda_i L + \cos \lambda_i L}{\sinh \lambda_i L + \sin \lambda_i L} \right) . \\ . (\sinh \lambda_i r - \sin \lambda_i r) \end{array} \right] \quad (12)$$

For finding mode shapes, the value of λi is to be calculated. As shear force at the free end of the beam is zero, the equation (12) is transformed into following equation (13) form

$$1 + \cos \lambda_i L \cos \lambda_i L = 0 \qquad (13)$$

NATURAL FREQUENCIES OF THE BEAM

The natural frequency of the beam depends on the Young's Modulus of the beam, moment of inertia of the beam, mass density of the beam, cross sectional area of the beam and λi which is a factor depending on mode shape of the beam. The natural frequency of the beam is given by

$$\omega_1 = \sqrt{\frac{EI}{\rho A}}\lambda_i^2 \qquad (14)$$

The natural frequencies using ANSYS software are determined to compare the analytical values determined using Equation (14). The first natural frequencies of the beam are given in Tab. 2.

Mode	λ	Theoretical ω (Hz)	ANSYS ω (Hz)
1	2,43	7,959	7,891
2	6,059	49,488	49,450
3	10,137	138,522	138,45
4	14,184	271,50	271,31

Tab. 2 *Comparison of four natural frequencies*

STATE SPACE REPRESENTATION

From equation (11), the state is represented by matrix form

$$x(t) = [q_1(t), q_1'(t), ..., q_N(t), q_N'(t)]^T \qquad (15)$$

When the controller is not present in the system, then the controller term will be zero. Then the state space equations are given as follows.

$$x'(t) = Ax(t) + B_f F(t) \qquad (16)$$

$$y(t,r) = C(r)x(t) \qquad (17)$$

$$y'(t,r) = D(r)x(t) \qquad (18)$$

When the controller is present in the system then the state space equations are given as follows.

$$x'(t) = Ax(t) + BU(t) + B_f F(t) \qquad (19)$$

where y (t) and y' (t) are displacement at the free end of the beam and velocity at the free end of the beam respectively.

$$A = \begin{bmatrix} 0 & 1 & & & \\ -\omega_1^2 & -2\xi_1\omega_1 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & 0 & 0 \\ & & & -\omega_N^2 & -2\xi_N\omega_N \end{bmatrix}$$

$$B = \frac{C_a}{\rho A L^3}\left[0 \quad \phi_1'(r_1) - \phi_1'(r_2) \quad . \quad . \quad 0 \quad \phi_N'(r_1) - \phi_N'(r_2)\right]^T$$

$$B_f = \frac{1}{\rho A L^3}\left[0 \quad \phi_1(r_f) \quad . \quad . \quad 0 \quad \phi_N(r_f)\right]^T \qquad (20)$$

$$C = \left[\phi_1(r) \quad 0 \quad . \quad . \quad \phi_N(r) \quad 0\right]$$

$$D = \left[0 \quad \phi_1(r) \quad . \quad . \quad 0 \quad \phi_N(r)\right]$$

For controlling external disturbances, control voltage given to actuating piezoelectric patch and corresponding gain matrix are represented in state space form by the following equation (21).

$$U(t) = -Kx(t) \qquad (21)$$

Substituting equation (21) in equation (19), we get,

$$x'(t) = Ax(t) - BKx(t) + B_f F(t) \qquad (22)$$

$$x'(t) = A_C x(t) + B_f F(t) \qquad (23)$$

MODAL DAMPING

Damping ratio is found from the transient analysis using ANSYS 7.0. Table 3 shows the modal damping values for the first four modes. Modal damping for the i th natural frequency = 2 ζ_i ω_i (24)

Mode	Logarithmic Decrement (ln (δ))	Damping ratio for the ith mode (ζi)	Natural frequency for the ith mode (ωi) rad/sec	Modal damping for the ith mode (2ζi ωi) rad/sec
1	0,09639	0,0153	50,00787	1,5302
2	0,1381	0,0229	310,9442	14,2412
3	0,104	0,0179	870,3592	31,1589
4	0,164	0,0252	1704,031	85,8832

Tab. 3 *Damping ratio and Modal damping of the beam*

OPTIMAL CONTROLLER DESIGN

Optimal controller called linear quadratic regulator is considered, which is having a quadratic cost function of states and controls. The formulation of the linear quadratic regulator for a linear system is as follows

$$x' = Ax + Bu \qquad (25)$$

$$y = Cx \qquad (26)$$

A control function u (t) has to be found that will minimize the cost function, J given by the equation

$$J = \int_0^\infty \left\{ x(t)^T Qx(t) + u(t)^T Ru(t) \right\} dt \qquad (27)$$

If R is very large relative to Q, which implies that the control energy is penalized heavily, the control effort will diminish at the expense of larger values for the state. When Q is very large relative to R, which implies that the state is penalized heavily, the control effort rises to reduce the state, resulting in a damped system. LQR problem corresponds to the following cost function equation (27) which is related to minimizing the total energy of the beam

$$J = \int_0^\infty \begin{bmatrix} \frac{1}{2}\int_0^L y'(t,r)^2 \rho A \, dr + \\ +\frac{1}{2}\int_0^L E_b I y''(t,r)^2 \, dr + \\ +R\, u(t)^2 \end{bmatrix} dt \qquad (28)$$

$$Tk = \frac{1}{2}\int_0^L y'(t,r)^2 \rho A \, dr \qquad (29)$$

$$Ve = \frac{1}{2}\int_0^L E_b I y''(t,r)^2 \, \rho \, A dr \qquad (30)$$

where Tk is the kinetic energy and Ve is the potential energy of the beam. By using the orthogonal condition, the cost function transformed is of the following form.

$$J = \int_0^\infty \left\{ x(t)^T Qx(t) + u(t)^T Ru(t) \right\} dt \qquad (31)$$

$$\text{where } Q = \frac{\rho A L^3}{2} diag\left(\omega_1^2, 1, \ldots, \omega_N^2, 1\right) \qquad (32)$$

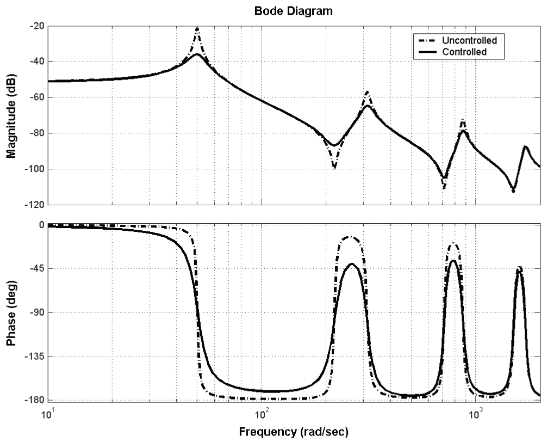

Fig. 3 *Bode plot representing transfer function between deflections at free end step input*

Total energy of the beam is considered as the cost function. The objective of this is to minimize the total energy of the beam. Fig. 3 shows the Bode plot representing transfer function between deflections at free end and step input given by F (t) =1 N for t ≥0. The response of the beam is shown in Figs. 4, 5. It is clearly understood that controlled vibrations are having less amplitude in tip deflection and velocity and settling time (1,5 sec) when compared with uncontrolled system (8 sec).

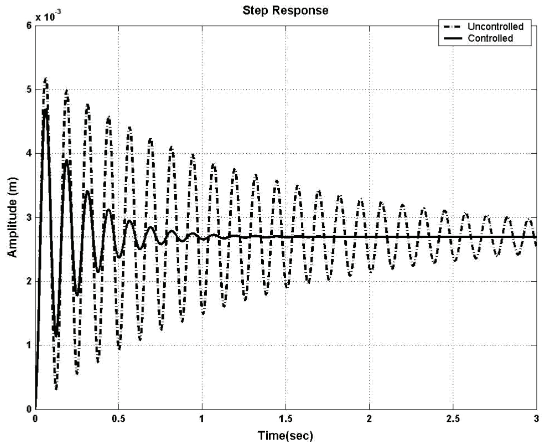

Fig. 4 *Response at the free end of the beam due to step input at free end*

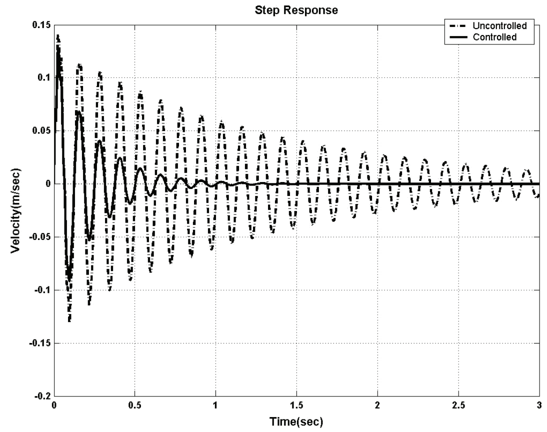

Fig. 5 *Velocity at the free end of the beam due to step input at free end of the beam*

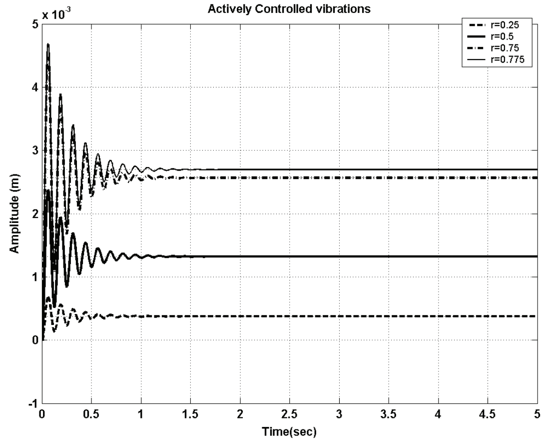

Fig. 8 *Response at different positions along the length of the beam due to step input*

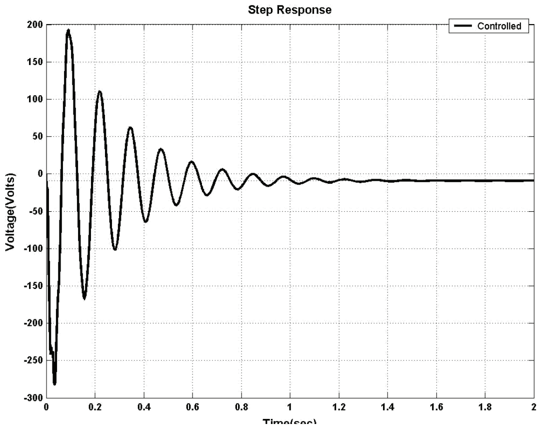

Fig. 6 *Voltage required for controlling the free end vibrations due to step input*

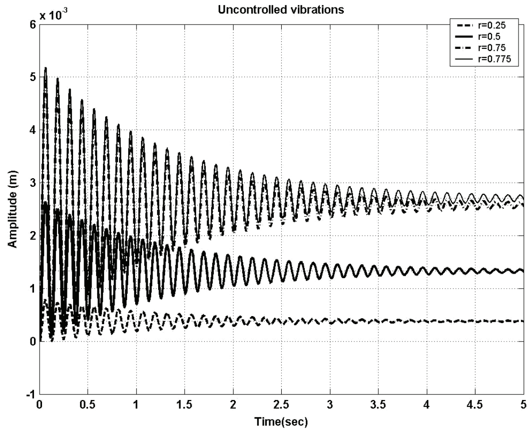

Fig. 7 *Response at different positions along the length of the beam due to step input*

Fig. 6 shows the variations of voltage required to control the vibration actively. The variations are achieved by optimal controller. Figs. 7 and 8 show responses at different positions of PZT actuator along the length of the beam due to step input for uncontrolled and controlled systems respectively. When the actuator is located near the fixed end, it observed that the vibrations are controlled more effectively than with other locations. At the same time, by varying control energy (R), the vibration settling time are determined and tabulated in Tab. 4. When R is decreased, the settling time reduced significantly.

[R]	With controller time required for vibration decay
[$4x10^{-4}$]	8
[$4x10^{-6}$]	7
[$4x10^{-8}$]	1,499
[$4x10^{-10}$]	0,1991
[$4x10^{-12}$]	0,1377

Tab. 2 *Comparison of four natural frequencies*

CONCLUSIONS

In this work, the vibrations caused by step input on flexible structure (cantilever beam) are actively controlled by using piezoelectric actuators. Vibrations are actively controlled at the free end of flexible structures using piezoelectric actuators. Vibration control is more effective when the actuator is placed at the root. Optimal linear quadratic regulator designed for active vibration control based on the cost functions related to minimization of total energy of the beam is proved to be an effective tool to control the vibrations actively. If state weighting matrix is very high, the vibration controlling effect is very high. Also, controller weighting matrix value should be very low for effective active vibration control. However, the cost implications have to be studied further.

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