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Experimental Analysis of Complex Chaotic System by Hilbert-Huang Transform Usage

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ABSTRACT

The paper demonstrates one promising algorithm for adaptive prediction of trajectory transitions between local basins of attraction of deterministic chaotic systems using Hilbert-Huang Transform. The expected transitions of higher dimensional chaotic systems are predicted by low order intrinsic modal functions, obtained from state variables by HHT. The behavior of chaotic systems in the state-space is transformed to system behavior in an approximated parameter-space obtained by Huang algorithm. Also a brief comparison to adaptive method using quadratic neural unit (QNU) with forcing inputs, introduced by Bukovský [2], is shown.

KEY WORDS

Lorenz's attractor, Hilbert-Huang Transform, adaptive prediction, deterministic chaotic systems.

INTRODUCTION

Complex and chaotic systems of higher order are the most challenging problem to research of control systems. There are several possibilities how to understand them. One of these is method developed by Bukovsk et al [2] and shown later. Our new method will be explained based on Lorenz attractor example [4]. The Lorenz attractor was postulated by Edward Lorenz in 1963. It is defined by a system of non-linear three-dimensional dynamic equations defined as follows:

$$\begin{aligned}\frac{\partial x}{\partial t} &= \sigma(y-x),\\ \frac{\partial y}{\partial t} &= x(r-z)-y,\\ \frac{\partial z}{\partial t} &= xy-bz.\end{aligned} \tag{1}$$

These equations represent physical meaning of forced convection in atmosphere. Variable σ is the Prandtl constant and r represents Rayleigh number. Common values in atmosphere, $\sigma = 10$, $r = 28$ and $b = 8/3$ produce chaotic behavior with randomly rotation changes. The solution of equation set is well known ”butterfly” pattern. The butterfly in 3-D is shown in Fig. 1.

Long time divergence is visible in Fig. 1. This case is called as ”strange attractor”. W. Tucker proved that ”strange attractor” is a fractal. Grassberger (1983) estimated its Hausdorff dimension as 2.06 ± 0.01. Correlation dimension was estimated to 2.05 ± 0.01.

Lorenz attractor is a case of well examined example of ”strange attractor” and we are able to describe its basic features as follows:

- ”Strange attractor” consists of continuous curve in phase space. The curve starts at known starting point and its endpoint is undefined. It means, the final length is not defined. The whole curve fits in a well defined region in phase space. It never crosses bounds of this region.

- It never intersects, copies or repeats.
- ”Strange attractor” has all features of fractals. It means the structure is repeating in scales.
- The curve flow in space is random, chaotic and unpredictable.

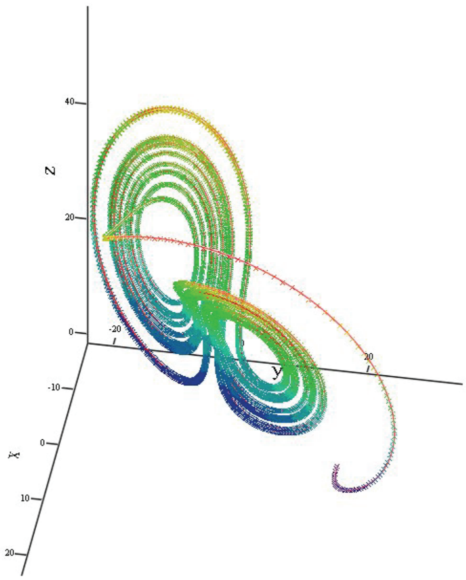

Fig. 1: *3-D Lorenz attractor solution.*

NON-LINEAR DERERMINISTIC DYNAMIC SYSTEM

Lorenz system is used as an example of non-linear chaotic system. We can compute the trajectory from big amount of known points, but we are not able to predict it in general. It means we are not able to predict when attractor goes to turn over. In this meaning there is only possibility to find attractor branches, but not to predict them. Evaluation is possible e.g. using variable y. We can see that variable y changes its sign between both branches. Value $y > 0$ represents right branch and $y < 0$ is representing the left one. The achievement of $y = 0$ value we can understand as a moment of turn-over. For better understanding have a look at Fig. 2.

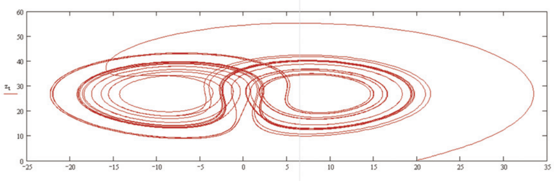

Fig. 2: *Branches* $y > 0$ *a* $y < 0$.

Let us have the following task: we have non-linear chaotic deterministic dynamic system whose parameters are completely unknown. We only know its behavior for several times in the past. Based on this incomplete information we need to predict turn over moments.

We can demonstrate an appropriate solution by using Lorenz system. Let us solve this problem using time dependency of $z(t)$, as displayed in green line in Fig. 3. Values $x(t)$ or $y(t)$ could be chosen too, but as you can see from Fig. 3, prediction of $z(t)$ is more problematic.

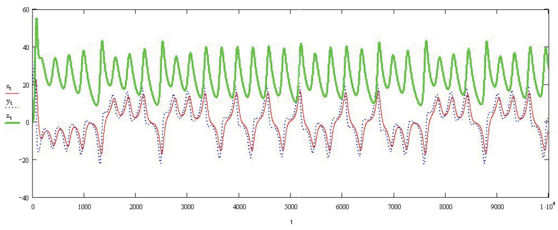

Fig. 3: *Time lines of state variables* x, y, z.

Before we start, it is good to note that every common method is failing in said prediction. We can take $z(t)$ from Fig. 3 as an example. It looks like sine wave at the first look and so it can mislead us to use Fourier transform for analysis.

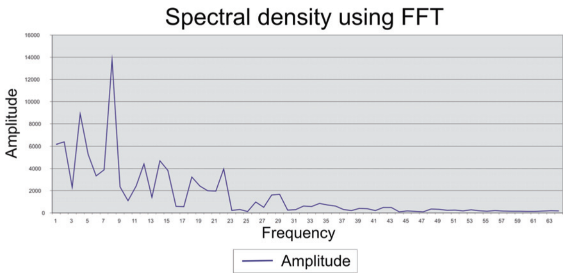

Fig. 4: *Spectral denstity of curve* $z(t)$ *from Fig. 3.*

Should we apply Fourier transform at our signal; we would receive from 9 to 10 dominant frequencies. The result is displayed on Fig. 4. This analysis was done via Excel (including the phase, which is not displayed here. Other important point is that its amplitude is not normalized).

It is evident that FT cannot be used for predicting of turn-over points. Inverse transformation IFT results in a course that is completely different from original $z(t)$. The main problems are linearity and stationary. Fourier transform is designed for linear and stationary a system, which is not the case of Lorenz system. Lorenz attractor is completely failing in this meaning.

HILBERT-HUANG TRANSFORM

At the first we have to notify Hilbert-Huang transform is not transform in the common mathematical meaning. Hilbert-Huang transformation can be better described as a semi-empiric algorithm. Whole process consists from two main steps. In the first step we reduce any given (measured) data into a collection of intrinsic mode functions (IMF) using the *empirical mode decomposition method* (EMD). In the second step we are able to apply Hilbert transform to all obtained intrinsic mode function components.

The main procedure of extracting IMFs is called *sifting*. The sifting is an iterative procedure. We obtain intrinsic mode functions (IMFs) as the result of this procedure.

Measured (input) data represent a wideband signal, which is very hard to understand. Sometimes it is even impossible. This is the reason why HHT is using sifting process. Sifting in first step of HHT extracts from signal relatively narrow-band IMF functions, which can improve understanding to the whole system. More detailed explanation you can find in related works [1], [5], [7] and [6].

For decomposition of signal $z(t)$ is used own sifting program. Routine been used for some time and it will be provided to anybody on request. In case of $z(t)$ in Lorenz attractor, the whole signal was decomposed to only two IMFs plus residues, while standard deviation did not come over value 0.16. All results are shown in Fig. 5 and 6.

IMF0 is displayed in Fig. 5 by bold blue line. Value oscillates around zero (note that Y axis is on the right side) and copies oscillation of input signal $z(t)$. $z(t)$ is shown by a thinner line. IFM0 is changing too much to be predicted.

We decided to use the next function *IMF1*. *IMF1* is displayed in Fig. 6 in the same manner as previous one.

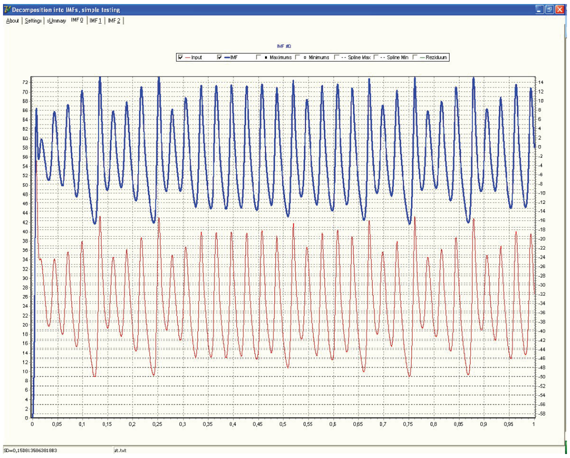

Fig. 5: *HHT result – IMF0 function.*

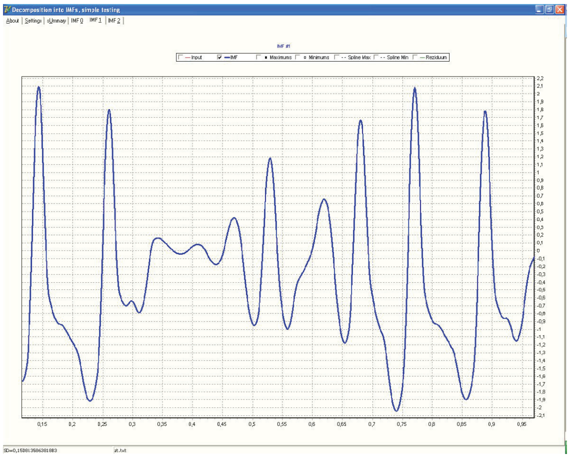

Fig. 6: *HHT result – IMF1 function.*

Idea which led us to prediction of whole process is following: Lorenz system is turning over between both branches. This change is systematic. That means this change should be hidden somewhere in the history of process. In fact, that we do not see anything on $z(t)$ curve flow, does not imply that this kind of information does not exist inside. Interesting question is **whether this hidden information comes from $z(t)$ into corresponding IMFs**, when sifted by HHT?

Turn over moment between both Lorenz branches is characterized by zero crossing of variables y and x. As is visible from Fig. 3, this is

Table 1: *Zero crossing times.*

Zero crossing time (×10 000)										
y+	1358	2533	3684	4289	4895	5794	6399	7297	7639	8808
y−	2196	3381	3987	4594	5214	6100	6728	7364	8471	9659
IMF+	1305	2495	3305	3920	4515	5170	5990	6680	7595	8770
IMF-	1555	2725	3680	4215	4825	5435	6340	6950	7830	9020
err %	4%	2%	10%	9%	8%	11%	6%	8%	1%	0%
err %	29%	19%	8%	8%	7%	11%	6%	6%	8%	7%

not applicable to $z(t)$. But IMF extracted from $z(t)$ are oscillating around zero level. Let us try to compare zero crossing moments of IMF and turn-over moments of Lorenz attractor. For more details look at Table 1. On lines $y+$ and $y-$ are zero crossing times for variable $y(t)$, found by numerical solution. Row "$y+$" shows times of cossing zero from negative to positive, row "$y-$" shows times of crossing zero from positive to negative direction. Be aware that time is multiplied by constant 10 000.

On lines *IMF+* and *IMF-* are time crossing values of empirical mode function *IMF1*. Time is evaluated from the chart.

Interesting moment is coming from this chart. There is eye-catching accordance between variable $y(t)$ and *IMF1* and it is clearly visible. Zero crossing moments of $y(t)$ and ıIMF1 seem to be equal. The difference is significant only at the very beginning. The general difference is around 8%. Error of up to 29% at the start is caused by border effects of sifting algorithm of HHT.

QNU (QUADRATIC NEURAL UNIT) AS COMMON OSCILLATORY UNIT

Similar prediction was done by Bukovský and company by using different kind of method. In this case, quadratic neural units (QNU) were used. Two dominant frequencies were chosen as the input. These frequencies were represented in Lorenz system flow. Neural network was trained to Lorenz attractor behavior with specific values. Numbers of values were similar as we did use in our case mentioned above. The result of the experiment discovered dramatic changes of QNU weights before turn over point. These changes are coming close to time of real one. Based on this fact these changes could be taken as type of prediction too.

Adaptive method introduced by Bukovský has advantage because of practical verification. Quadratic neural units could be used for prediction of higher order systems. The disadvantage of this method in real time usage could be parameters setup. These parameters need to be defined for the following system adoption. This adoption has to be done as the first before real time usage. Other disadvantage could be parameter of QNU detection sensitivity.

HHT method is easier for evaluation based on example. Acceptable results are coming from? But we need to be aware around border effects. These effects still do not allow a real time usage of this method. Acceptable results are coming only in the middle of time interval. For real time usage we need to evaluate historical data as first point. IMF functions can be evaluated then. In this case results of HHT method become similar to quadratic neural unit training.

CONCLUSION

New hopeful method is mentioned and described in this article. This method could help in behavior prediction of non linear deterministic dynamic system. Based on example there are explained steps of usage and its principle. HTT method results are evaluated against the results of QNU one.

Main disadvantage of HHT method usage are unsolved questions around border conditions. We intend to solve this issue in the future.

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