Acta Mechanica Slovaca 13 (4): 68-73, 2009
DOI: 10.2478/v10147-010-0039-8

Virtual Machining and its Experimental Verification

Peter DEMEČ (SK) peter.demec@tuke.sk
Jozef SVETLÍK (SK) jozef.svetlik@tuke.sk

ABSTRACT

Research, verification and implementation of various methods of rapid prototyping are getting more intensified in the recent time. One of possible rapid prototyping techniques is a virtual machining which allows testing the properties of designed machine tool using its mathematical model. This article presents the example of virtual machining on a virtual small centre lathe and experimental verification of obtained results.

KEY WORDS

machine tools, virtual prototyping, virtual machining.

THEORETICAL FUNDAMENTS

There are basic mathematical relations developed in [1] for modeling of working accuracy of machine tools with serial kinematics structure in common form. Mathematical model of ideal machined surface is in matrix equation

$$r_0(t) = \left(\prod_{i=1}^{n-1} R_{i,i-1}(t)\right) \cdot r_n + \sum_{i=1}^{n-1}\left[\left(\prod_{j=i}^{i} R_{j,j-1}(t)\right) \cdot (T_{i+1,i}(t) + K_{i+1,i})\right] + T_{10}(t) + K_{10}, \tag{1}$$

where

$r_0(t)$ – is position vector of active tool's point according to part's coordinate system,

r_n – is position vector of active tool's point according to tool holder's coordinate,

$R_{i,i-1}(t)$ – is Transformed matrix of rotation motion of modeled body T_i around of one coordinate axis of model body T_{i-1},

$T_{i+1,i}(t)$ – is Transformed vector of straight motion of modeled body T_{i+1} in direction of one coordinate axis of model body T_i,

$K_{i+1,i}$ – is vector of start position of coordinate system of model body T_{i+1} in coordinate system of model body T_i,

t – is time.

The final machining inaccuracy is determined by sum of part deformation due to production forces and position inaccuracy of all modeled bodies (machine nodes) from tool to machined part in coordinate system of machined part in time that in mathematical language could be write in a form

$$\Delta(t) = \Delta_0(t) + \sum_{i=1}^{n}\left[\left(\prod_{j=i}^{i} R_{j,j-1}(t)\right) \cdot (\delta_i(t) + \varepsilon_i(t) \cdot r_i(t))\right], \tag{2}$$

where deformation vector of machined part is

$$\Delta_0(t) = \delta_0(t) + \varepsilon_0(t) \cdot r_0(t) \tag{3}$$

and vector

$$\Delta_i(t) = \delta_i(t) + \varepsilon_i(t) \cdot r_i(t) \tag{4}$$

represent the final inaccuracy of active tool's point position caused by inaccuracy of model body T_i expressed in coordinate system model body itself.

The final inaccuracy of active tool's point position caused by inaccuracy of model body T_i, expressed in coordinate system of work piece is contained in form (2) too, in bracket after sum and it is function

$$\Delta_{i,0}(t) = \left(\prod_{j=i}^{i} R_{j,j-1}(t) \right) \cdot \Delta_i(t). \tag{5}$$

The vector of linear inaccuracies of model body Ti is defined by relation

$$\delta_i(t) = \left[\delta_{xi}(t), \delta_{yi}(t), \delta_{zi}(t)\right]^T, \tag{6}$$

where $\delta_{xi}(t), \delta_{yi}(t), \delta_{zi}(t)$ are linear inaccuracies in directions of corresponding coordinate axis.

The matrix of angular inaccuracies of model body T_i is defined by relation

$$\varepsilon_i(t) = \begin{pmatrix} 0 & -\Psi_i(t) & \upsilon_i(t) \\ \Psi_i(t) & 0 & -\varphi_i(t) \\ \upsilon_i(t) & \varphi_i(t) & 0 \end{pmatrix}, \tag{7}$$

where $\varphi_i(t), \upsilon_i(t), \Psi_i(t)$ are angular inaccuracies (of rotation about axes X_i, Y_i, Z_i).

The position vector of active tool's point in coordinate system of model body T_i – vector $r_i(t)$ in equations (2) and (4) is defined by relation

$$\begin{aligned} r_i(t) = {} & \left(\prod_{j=i}^{n-1} R_{j+1,j}(t) \right) \cdot r_n \\ & + \sum_{j=i}^{n-2} \left[\left(\prod_{k=i}^{j} R_{k+1,k}(t) \right) \cdot \left(T_{j+2,j+1}(t) + K_{j+2,j+1}\right) \right] \\ & + T_{i+1,i}(t) + K_{i+1,i}. \end{aligned} \tag{8}$$

The described mathematical model we used in virtual machining – simulation of machined virtual part on virtual machine.

Numerical experiments start with creating of *computational model of designed machine*, whereby we determine sequences of movable and immovable modeled bodies of machine in direction from work piece to tool, define their coordinate systems, define mathematically their mutual starting position of modeled bodies, motions of individual modeled bodies of computational model and define transformed matrix and vectors of modeled bodies (in common form), it is necessary to the equation (1).

Next step is design of virtual model of *machined part* and its mathematical definition. This virtual model will be ”machined” on computer model of researched machine tool with aim to find out the working inaccuracy with certain probability. Therefore we propose the model of a machined part as the body of simple geometric form with dimensions close to maximum values suitable for the corresponding machine tool. The same is true for material of the machined part. We choose the material which needs the maximal cutting forces for corresponding machine tool.

Machining of modeled part on mathematical model of researched machine tools we are proposing on the same steps. It is suitable to model the machining of the surfaces that are typical for corresponding machine tool. When conditions of the machining are done we must again redefine the transformed matrices and vectors in computational model of machine tool.

The process of virtual machining itself is suitable to realize in two stages. In first stage we will not take into account the effect of cutting forces. By means of matrix equation (1), which in fact represents *mathematical model of ideal machined surface* in common form, we calculate for different values of time $t \in \langle 0; T \rangle$ components of position vectors $r_i(t)$ according to (8) and $r_0(t)$ according to (1), while the T is the time needed for complete surface machining with corresponded machining parameters (feed and spindle speed).

In the second stage we include into calculations the influence of acting forces, creates some inaccuracies of positions of modeled bodies, that are able to describe their positions in any time $t \in \langle 0; T \rangle$ by equations (2) to (7). The result is mathematical *model of real machined surface* of virtual work piece. We get that as sum of relations values (1) and

(2) in corresponding time t.

The real machined surface we can mathematically express by equation

$$S(t) = r_0(t) + \Delta(t) \quad (9)$$

ANALYSIS MODEL

Now we can show the practical sample of virtual machining of virtual part on virtual machine tool – model of little lathe EMCO PC TURN 50. Virtual 3D model realized in Solid Edge environment of the machine tool is on the Fig. 1 and its calculation model is illustrated on Fig. 2. The lathe is replaced by six modeled bodies: T_1 – spindle, T_2 – headstock, T_3 – bed, T_4 – bed tool head slide, T_5 – traverse tool head slide, T_6 – turret head with tool. For virtual machining we use cylinder work piece with length L_p = 250 mm, diameter $\varnothing D_p$ = 35 mm. Cutting depth for machining of virtual work piece a_p = 1 mm, feed for one rev of spindle f_Z = 0,08 mm, whereby machining by movement longitudinal slide of support in direction into the headstock without supported with tailstock.

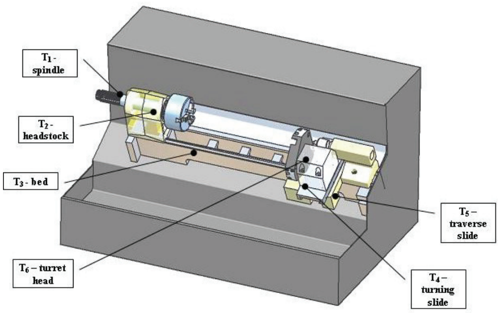

Fig. 1: *Virtual 3D model of lathe.*

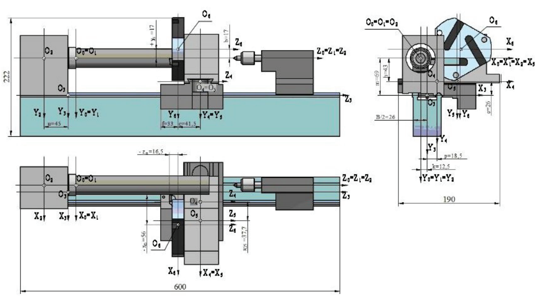

Fig. 2: *Analysis model of lathe.*

Turret head, support (and its longitudinal even cross slide), bed and headstock are not move each other rotation motions and spindle towards work piece, transformation matrices of rotation motions of these modeled body will be unit, so

$$R_{65}(t) = R_{54}(t) = R_{43}(t) = R_{32}(t) = R_{10}(t) = E. \quad (10)$$

Spindle against headstock doing rotation motion that is the reason, why correspondent transform matrix of mutual rotational moving of these two modeled bodies will be this form

$$R_{21}(t) = \begin{pmatrix} \cos\gamma_2(t) & -\sin\gamma_2(t) & 0 \\ \sin\gamma_2(t) & \cos\gamma_2(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (11)$$

where $\gamma_2(t)$ is immediate angle of revolution spindle against headstock about Z axis, which calculate on the base knowing operating rotation speed of spindle n_v and time from form

$$\gamma_2(t) = \frac{\pi \cdot n_v \cdot t}{30} \quad [\mathrm{rad}]. \quad (12)$$

Turret head, longitudinal even cross slide of support and bed too, headstock, tailstock and work piece is each other do not move linear moving, that is the reason why transformation vectors of modeled bodies of virtual lathe will be zero. It possible makes this mathematical form

$$T_{65}(t) = T_{54}(t) = T_{32}(t) = T_{21}(t) = T_{10}(t) = 0. \quad (13)$$

Longitudinal slide of support is in line moving with bed in direction of negative axis Z that is the reason, why transform vector of relative line moving of these two modeled bodies will be in form

$$T_{43}(t) = [0, 0, -s_{Z4}(t)]^T = \left[0, 0, -f_Z \cdot n_v \cdot \frac{t}{60}\right]^T. \quad (14)$$

Where $s_{Z4}(t)$ is immediate trajectory of longitudinal slide support (depend on size of translation f_Z [mm.rev^{-1}], rotational speed of spindle n_v [min^{-1}] and time t [s]).

Vectors of start positions of coordinate systems of modeled bodies T_{i+1} in coordinate systems modeled bodies T_i depend on machine design and from location of tool at the start of virtual machining process. For research case have these vectors items in common formulations (look at Fig. 2), let us say after induction of concrete numerical values (expressions in millimeters) next

$$K_{65} = [a, -(b+h), -c]^T = [7,5; -60; -41,5]^T,$$
$$K_{54} = [x_{05}, 0, 0]^T = [17,5 + R_p - a_p, 0, 0]^T = [34; 0; 0]^T,$$
$$K_{43} = [p, -e, z_{04}]^T = [18,5; -26; 323]^T,$$
$$K_{32} = [k, m, u]^T = [12,5; 69; 45]^T,$$
$$K_{21} = [0, 0, -n]^T = [0, 0, -60]^T,$$
$$K_{10} = [0, 0, 0]^T = 0. \tag{15}$$

Positional vector of function point tool in coordinate system of – modeled body T_6 is

$$r_n = r_6 = [-56; 17; -16,5]^T. \tag{16}$$

After substitution correspondent transformational matrix and vectors to the matrix equation (1) and correspondent mathematical modifications we get mathematical model of ideal finish machined surface of virtual work piece in form

$$r_0(t) = \begin{pmatrix} \cos\gamma_2(t) & -\sin\gamma_2(t) & 0 \\ \sin\gamma_2(t) & \cos\gamma_2(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 16,5 \\ 0 \\ 250 - f_z \cdot n_v \cdot \frac{t}{60}. \end{pmatrix} \tag{17}$$

NUMERICAL EXPERIMENT AND HIS VERIFICATION

Like writhed above, modeled work piece (Fig. 3) was designed like cylinder body long L_p = 250 mm and diameter $\varnothing D_p$ = 35 mm. Depth of cutting for machining of virtual work piece was choices a_p = 1 mm, feed per revolution of spindle f_Z = 0,08 mm, it will machining by moving of lengthwise slide in direction to the headstock without underpinned work piece by tailstock. Material of work piece – steel 12 050.1 (STN, what corresponded steel C45 in DIN standard), way of machining – roughing be taking-off knife with hard alloy plates P30 (ISO standard) form SNMM 120408 without cooling. Correspondent angles and radius of shape cutting knife is: γ_0 = -6°, α_o = -6°, λ_s = -6°, κ_r = 75°, r_ε = 0,8 mm, rotational speed of spindle n_v = 1400 min^{-1} (on radius of machining R_p = 16,5 mm equivalent is cutting speed v_c = 145,142 m.min^{-1}).

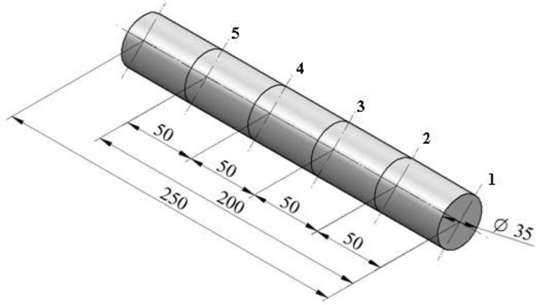

Fig. 3: *Model of work piece.*

For above-mentioned conditions, we can calculate particular components of cutting force (cutting resistance) from structural equations, which can be found in literature (e.g. [2]). In order to minimize calculation inaccuracies, we investigated (*determined*) these structural equations for concrete machined material and tool experimentally, in cooperation with the Department of Machining and Production Technology of Faculty of Mechanical Engineering of University of Žilina. For measurement, we used the dynamometer KISTLER type 9441, 3-channel charge amplifier KISTLER model 5006 and computer with digital-analog converter PCL 818 HG. Results of measurement were processed by software DASYLAB v.3.1. Experimentally obtained structural equations are in following form

$$\begin{aligned} F_c &= 993 \cdot a_p^{0,87} \cdot f^{0,54} \quad [\text{N}], \\ F_f &= 410 \cdot a_p^{0,41} \cdot f^{0,46} \quad [\text{N}], \\ F_p &= 361 \cdot a_p^{1,21} \cdot f^{0,25} \quad [\text{N}], \end{aligned} \tag{18}$$

where we substitute displacement and cutting depth values in millimeters. When we substitute numerical values based on chosen conditions of machining into the equations (18), corresponding values of cutting force components are as follows: F_c = 253,9 N, F_f = 128,3 N and F_p = 192 N. These values were used in calculations of deformations of modeled bodies and work pieces, which are necessary to determine corresponding inaccuracies.

Numerical experiment was realized in Excel 2003 at design and technological conditions specified above. Virtual work piece is slightly fixed in a chuck with front end of work piece on axe $z_0(0)$ = 250 mm from the zero point of work piece coordinate system. Machining of 200mm long cylindrical surface at rotation speed of spindle and saddle traverse specified above will take 107,14286 sec., while time of one spindle revolution is 0,04286 sec. As from the practical viewpoint we do not need full digital model of whole work piece but only its cross-sections with such spacing that allows identifying e.g. deviations of circularity and cylindricity of finished virtual body, calculations were made this way

- Virtual machining of circle (include correspondent calculations) in one revolution of spindle on the coordinate $z_0(0)$ = 250 mm (position 1 on Fig. 3),
- 624 revolutions of spindle without calculating,
- Virtual machining of circle (include correspondent calculations) in one revolution of spindle on the coordinate correspondent 625-th revolution of spindle, i.e. $z_0(26,7857)$ = 200 mm (position 2 on Fig. 3),
- Etc. pending virtual machining of circle (include correspondent calculations) in one revolution of spindle on the coordinate correspondent 2500-th revolution of spindle, i.e. $z_0(107,14286)$ = 50 mm (position 5 on Fig. 3).

In every position from 1 to 5 (Fig. 3) it was via MKP realized computing of deformation of modeling body from T_1 to T_6 and work piece and correspondent results was implement into the forms from (3) to (7). By using of equation (1), (2), (8) and (9) it computed correspondent machined diameters on virtual machined modeled work piece in diameters 1 to 5 (Fig. 3). Like a demonstration of perfected static analysis in virtual static analyses in virtual machining we are showing on Fig. 4 results of computing of deformations of lathe slide by forced cutting resistance at machining of work piece in position 3. On Fig. 5 is graphical showing progress of change of real machining radius of virtual work piece in correspondent from position of tool on axis Z. Correspondent numerical value is listed in Table 1. Real machining work piece was measured on Department of Machining and Production Technology of Faculty of Mechanical Engineering of University of Žilina on 3D coordinate measurement machine TESA MICRO-HITE 3D DCC. In the sections 1 to 5 (according to Fig. 3) discovered real value of machined diameters, which is mentioned in Table 1 too. On a Fig. 6 are mentioned results of measurements in tool positions 1 and 2 in graphical form.

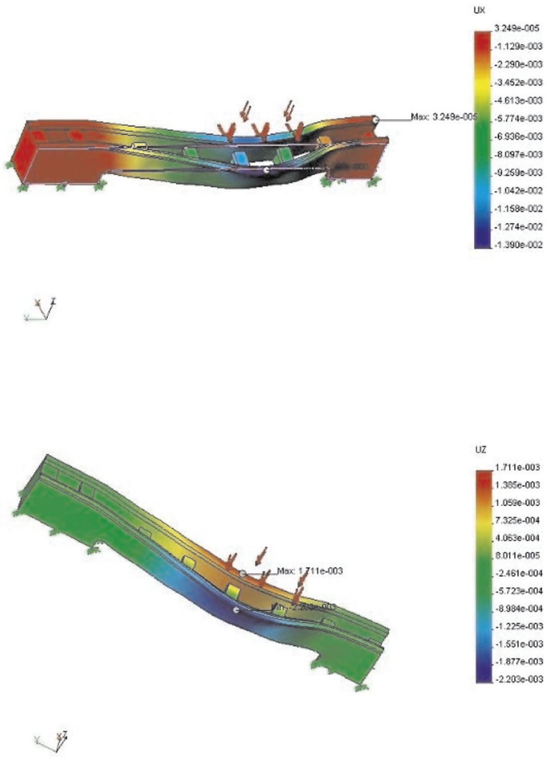

Fig. 4: *Static analysis of bed.*

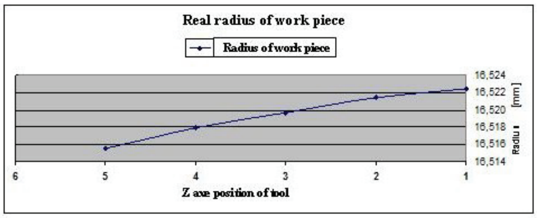

Fig. 5: *Real radius of virtual machined work piece.*

Table 1: *Comparison of results of virtual machined and experiment.*

Position of tool	1	2	3	4	5
Computed radius [mm]	16,5224	16,5214	16,5197	16,5179	16,5156
Computed diameter [mm]	33,0448	33,0428	33,0394	33,0358	33,0318
Diameter determined via experiment [mm]	33,468	33,466	33,461	33,457	33,451

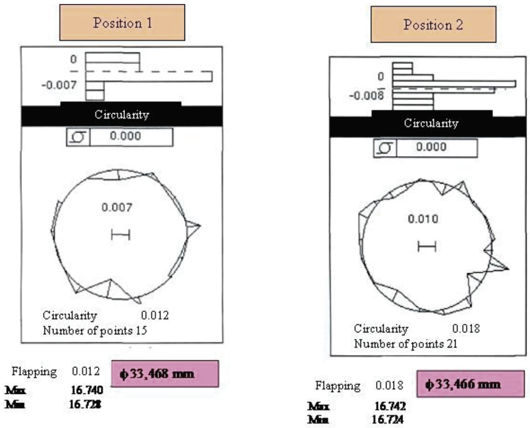

Fig. 6: *Measured results of real work piece.*

CONCLUSION

The comparison of the results of virtual and real machining shows that all measured diameters in all examined cross-sections are bigger than the results obtained analytically. In both cases (measurement and calculation) all measured diameters are bigger than ideal diameter of machined surface, which is 33mm. This situation follows from the principle of dimension deviations generated during the machining process. Action of cutting force components causes that tool mounted in a turret head and carried by a support is pushed away from a work piece (in positive X-axis direction) or depressed in the direction of positive Y-axis downwards. On the other side, work piece, headstock and spindle are pushed away in opposite directions of corresponding coordinate axes due to impact of components of cutting force. The result is that headstock centre tip is, in fact, machining the work piece on the bigger diameter than ideal. The fact that measured diameter values of work piece have been found bigger than calculated ones means that real machining creates bigger inaccuracy than found analytically (maximum error between calculated and measured value is -1,2655 %). It can be justified by the fact that each calculation includes certain inaccuracies following from simplifying the problem to be solvable. In this work we also haven't considered surface deformations, temperature and dynamic influences, etc.

ACKNOWLEDGEMENT

The paper is the result of solution of grant project VEGA number 1/4162/07 Research of working accuracy of machine tool in the phase of its virtual prototyping.

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