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2DOF Analog and Digital Controller Tunning for Integrating Plants with Time Delay

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ABSTRACT
The article describes the tuning of 2DOF analog and digital PI and PID controllers for integrating plants with a time delay. Two methods are considered. The first is the multiple dominant pole method, which is an analytical method and the second is an experimental method. Both methods are very easy to use and they give a relatively good quality to control processes. The application is shown in the example.

KEY WORDS
2DOF controllers, integrating plants, time delay controller tuning.

INTRODUCTION
Tuning PI and PID controllers for integrating plants belongs among the more demanding problems. It is caused by the existence of two integrators in the control loop, and thereby by the great predisposition to large overshoots, eventually to oscillation and stability loss [2] – [9].
In article [9] the tuning of 2DOF analog PI and PID controllers by the multiple dominant pole method for integrating plants with a time delay is described in detail. In this article the results are expanded for corresponding 2DOF digital controllers also. Furthermore, an experimental tuning method is presented.

CONTROL SYSTEM WITH DIGITAL CONTROLLER
In Fig. 1 the simple control system with the digital controller is shown, where: $G_C(z)$ is the digital controller transfer function, $G_P(z)$ – the plant transfer function, $W(z)$ – the transform of desired variable, $E(z)$ – the transform of the control error, $U(z)$ – the transform of the manipulated variable, $V(z)$ – the transform of the disturbance variable, $Y(z)$ – the transform of the controlled variable.
It is supposed that the quantization error is negligibly small and therefore the concepts discrete and digital are considered to be equivalent.

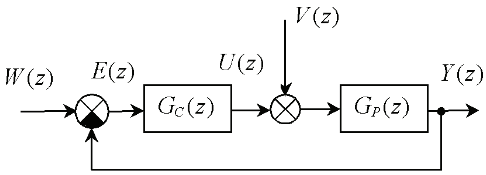

Fig. 1: *Control system with a standard digital controller.*

The integrating plant with a time delay with the transfer function

$$G_P(s) = \frac{k_1}{s} e^{-T_d s} \tag{1}$$

is considered.

Suppose that the digital-analog converter can be substituted by the sampler and the zero-order hold, and that time delay T_d is an integer multiple of the sampling period T, i.e. $T_d = dT$. In this case the discrete transfer function of the plant (1) has the form [7, 8]

$$\begin{aligned} G_P(z) &= \frac{z-1}{z} Z\left(L^{-1}\left(\frac{1}{s}G_P(s)\right)\bigg|_{t=kT}\right) \\ &= \frac{k_1 T}{z-1} z^{-d}. \end{aligned} \tag{2}$$

The transfer functions of the standard analog and digital PI and PID controllers are given in Tab. 1, where: r_0 is the controller gain, T_I - the integral time, T_D - the derivative time.

For simplicity the standard digital PI controller will be furthermore considered only, see Tab. 1.

The error transfer functions for the control system in Fig. 1 and plant (2) are given

$$\begin{aligned} G_{we}(z) &= \frac{E_w(z)}{W(z)} = \frac{1}{1+G_C(z)G_P(z)} \\ &= \frac{(z-1)^2}{(z-1)^2 + r_0 k_1 T\left[\left(1+\frac{T}{T_I}\right)z - 1\right] z^{-d}}, \end{aligned} \tag{3}$$

$$\begin{aligned} G_{ve}(z) &= \frac{E_v(z)}{V(z)} = -\frac{G_P(z)}{1+G_C(z)G_P(z)} \\ &= -\frac{k_1 T(z-1)}{(z-1)^2 + r_0 k_1 T\left[\left(1+\frac{T}{T_I}\right)z - 1\right] z^{-d}} z^{-d}. \end{aligned} \tag{4}$$

For step changes of the desired w_0 and disturbance v_0 variables the corresponding error transforms are given by formulas

$$\begin{aligned} E_w(z) &= G_{we}(z)W(z) \\ &= \frac{(z-1)w_0 z}{(z-1)^2 + r_0 k_1 T\left[\left(1+\frac{T}{T_I}\right)z - 1\right] z^{-d}}, \end{aligned} \tag{5}$$

$$\begin{aligned} E_v(z) &= G_{ve}(z)V(z) \\ &= -\frac{k_1 T v_0 z}{(z-1)^2 + r_0 k_1 T\left[\left(1+\frac{T}{T_I}\right)z - 1\right] z^{-d}} z^{-d}. \end{aligned} \tag{6}$$

On the basis of the final value theorem there can be obtained

$$\begin{aligned} e_w(\infty) &= \lim_{z\to 1}[(z-1)E_w(z)] = 0, \\ e_v(\infty) &= \lim_{z\to 1}[(z-1)E_v(z)] = 0. \end{aligned} \tag{7}$$

Just because it demands a zero steady state error caused by the disturbance step in the plant input it is necessary to use controllers with the integrating term.

Unfortunately, the integrating term causes some problems, from which the next considerations follow.

The control areas for step changes of the desired and disturbance variables in accordance with (5) and (6) are given by relations

$$T\sum_{k=0}^{\infty} e_w(kT) = T\lim_{z\to 1} E_w(z) = 0, \tag{8}$$

$$T\sum_{k=0}^{\infty} e_v(kT) = T\lim_{z\to 1} E_v(z) = -\frac{v_0 T_I}{r_0} \neq 0. \tag{9}$$

The interpretation of the obtained relations is very important. From first relation (8) it follows that the control area is equal to zero, i.e. for the servo step response the course of the controlled variable cannot be without the overshoot (the sum of the areas over the half line $w(t) = w_0$ is equal to sum of the areas under the half line).

The interpretation of second relation (9) is thereby the corresponding controller tuning and it is always possible to obtain the aperiodic regulatory response without the overshoot.

It is obvious that the same conclusions hold for the standard digital controller PID and corresponding analog controllers and arbitrary integrating plants.

From the above mentioned, it follows that the servo and regulatory step responses cannot be simultaneously aperiodic without overshoots for the integrating plants and for standard analog or digital PI or PID controllers.

The 2DOF digital PID controller can be described by the relation [7, 8]

$$\begin{aligned} U(z) = r_0 \Bigg[& bW(z) - Y(z) + \frac{T}{T_I}\frac{z}{z-1}E(z) \\ & + \frac{T_D}{T}\frac{z}{z-1}\Big(cW(z) - Y(z)\Big)\Bigg] \end{aligned} \tag{10}$$

Table 1: *Standard controller transfer functions.*

	TYPE	CONTROLLER	
		ANALOG	DIGITAL
1	PI	$r_0\left(1+\frac{1}{T_I s}\right)$	$r_0\left(1+\frac{T}{T_I}\frac{z}{z-1}\right)$
2	PID	$r_0\left(1+\frac{1}{T_I s}+T_D s\right)$	$r_0\left(1+\frac{T}{T_I}\frac{z}{z-1}+\frac{T_D}{T}\frac{z-1}{z}\right)$

and the corresponding 2DOF analog PID controller

$$U(s)=r_0\left[bW(s)-Y(s)+\frac{1}{T_I s}E(s)\right. \\ \left.+T_D s\Big(cW(s)-Y(s)\Big)\right], \tag{11}$$

where b is the set-point weight of the proportional term, c – the set-point weight of the derivative (difference) term.

For $b=c=1$ from relations (10) and (11) the standard digital and analog PID controllers can be obtained (row 2 in Tab. 1), for $b=1$ and $T_D=0$ the standard digital and analog PI controllers can be obtained (row 1 in Tab. 1).

DIGITAL CONTROLLER TUNING

For the 2DOF analog PI and PID controllers the values of the adjustable controller parameters were derived by the multiple dominant pole method in [9] and they are given in Tab. 2 for $T=0$. On the basis of the authors experience the adjustable parameter values of the analog PI and PID controllers were determined. These values give better results and they are given in Tab. 2 as well.

For determining the adjustable parameter values for the corresponding digital controllers the approximate method of the bilinear transform was used [1].

It is obvious that the following holds

$$s=\lim_{T\to 0}\frac{2}{T}\frac{z-1}{z+1}\bigg|_{z=e^{sT}} \tag{12}$$

i.e.

$$G_p'(s)\approx G_p(z)\bigg|_{z\approx\frac{1+s\frac{T}{2}}{1-s\frac{T}{2}}}\approx\frac{k_1(1-s\frac{T}{2})}{s}e^{-sT_d}. \tag{13}$$

After simple approximation

$$1-s\frac{T}{2}\approx e^{-s\frac{T}{2}}$$

the modified plant transfer function

$$G_p'(s)\approx\frac{k_1}{s}e^{-s(T_d+\frac{T}{2})} \tag{14}$$

can be obtained [7, 8].

The transfer function (14) includes the quasi behaviour of the digital-to-analog converter and expresses the fact that for a digital controller the same adjustable parameter values can be used like that for a corresponding analog controller but the time delay must be increased to half the time delay.

EXAMPLE

It is necessary to tune the analog and digital 2DOF PI and PID controllers for the integrating plant with a time delay and transfer function (the time delay is in seconds)

$$G_P(s)=\frac{0.1}{s}e^{-10s}.$$

Solution: For the given plant it holds that: $k_1=0.1\ \mathrm{s}^{-1}$; $T_d=10$ s.

For the choice of the sampling period it is recommended $T\leq 0.3T_d$ [7, 8], therefore the value $T=2$ s was chosen.

On the basis of the relations in Tab. 2 the adjustable parameter values were computed:

a) Multiple dominant pole method (MDPM)

Table 2: *Adjustable parameter values for the multiple dominant pole method (MDPM) and experimental method (EM).*

METHOD	CONTROLLER $\langle$ ANALOG $T=0$, DIGITAL $T>0$					
	TYPE	r_0^*	T_I^*	T_D^*	b^*	c^*
MDPM	PI	$0.461\frac{1}{k_1\ \ T_d+\frac{T}{2}}$	$5.828\ \ T_d+\frac{T}{2}$	–	0.293	–
	PID	$0.784\frac{1}{k_1\ \ T_d+\frac{T}{2}}$	$3.732\ \ T_d+\frac{T}{2}$	$0.263\ \ T_d+\frac{T}{2}$	0.423	0.634
EM	PI	$0.6\frac{1}{k_1\ \ T_d+\frac{T}{2}}$	$4.15\ \ T_d+\frac{T}{2}$	–	0.25	–
	PID	$\frac{1}{k_1\ \ T_d+\frac{T}{2}}$	$3\ \ T_d+\frac{T}{2}$	$0.35\ \ T_d+\frac{T}{2}$	0.3	0.3

Analog 2DOF PI controller ($T=0$)**:**

$$r_0^* = 0.461\frac{1}{k_1 T_d} \doteq 0.46,$$
$$T_I^* = 5.828 T_d = 58.28.$$

Digital 2DOF PI controller ($T=2$)**:**

$$r_0^* = 0.461\frac{1}{k_1\ \ T_d+\frac{T}{2}} \doteq 0.42,$$
$$T_I^* = 5.828\ \ T_d+\frac{T}{2}\ \ \doteq 64.11.$$

Analog 2DOF PID controller ($T=0$)**:**

$$r_0^* = 0.784\frac{1}{k_1 T_d} = 0.78,$$
$$T_I^* = 3.732 T_d = 37.32,$$
$$T_D^* = 0.263 T_d = 2.63.$$

Digital 2DOF PID controller ($T=2$)**:**

$$r_0^* = 0.784\frac{1}{k_1\ \ T_d+\frac{T}{2}} \doteq 0.71,$$
$$T_I^* = 3.732\ \ T_d+\frac{T}{2}\ \ \doteq 41.05,$$
$$T_D^* = 0.263\ \ T_d+\frac{T}{2}\ \ \doteq 2.89.$$

b) Experimental method (EM)

Analog 2DOF PI controller ($T=0$)**:**

$$r_0^* = 0.6\frac{1}{k_1 T_d} = 0.6,$$
$$T_I^* = 4.15 T_d = 41.5.$$

Digital 2DOF PI controller ($T=2$)**:**

$$r_0^* = 0.6\frac{1}{k_1\ \ T_d+\frac{T}{2}} \doteq 0.55,$$
$$T_I^* = 4.15\ \ T_d+\frac{T}{2}\ \ = 45.65.$$

Analog 2DOF PID controller ($T=0$)**:**

$$r_0^* = \frac{1}{k_1 T_d} = 1,$$
$$T_I^* = 3 T_d = 30,$$
$$T_D^* = 0.35 T_d = 3.5.$$

Digital 2DOF PID controller ($T=2$)**:**

$$r_0^* = \frac{1}{k_1\ \ T_d+\frac{T}{2}} \doteq 0.91,$$
$$T_I^* = 3\ \ T_d+\frac{T}{2}\ \ = 33,$$
$$T_D^* = 0.35\ \ T_d+\frac{T}{2}\ \ = 3.85.$$

In Fig. 2 and 3 the responses of the control system with the analog and digital PI controllers are shown. For 2DOF PI controllers the weights were chosen: $b = 0.293$ for the multiple dominant pole method (MDPM) and $b = 0.25$ for the experimental method (EM).

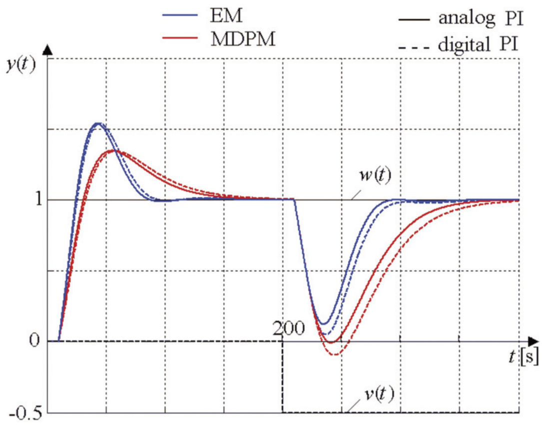

Fig. 2: *Responses of control system with analog and digital PI controllers for the multiple dominant pole method (MDPM) and experimental method (EM).*

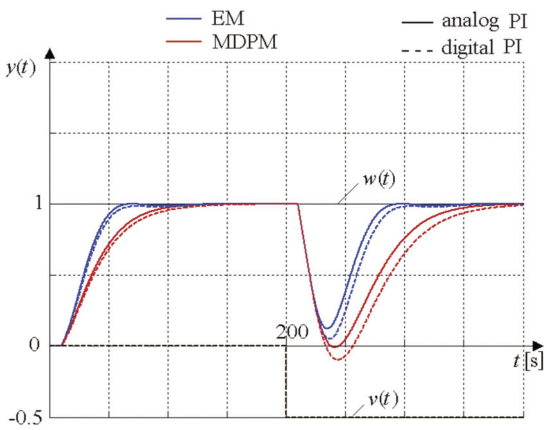

Fig. 3: *Responses of a control system with analog and digital 2DOF PI controllers for the multiple dominant pole method (MDPM) and experimental method (EM).*

In Fig. 4 and 5 the responses of the control system with the analog and digital PID controllers are shown. For 2DOF PID controllers the weights were chosen: $b = 0.423$, $c = 0.634$ for the multiple dominant pole method (MDPM) and $b = c = 0.3$ for the experimental method (EM).

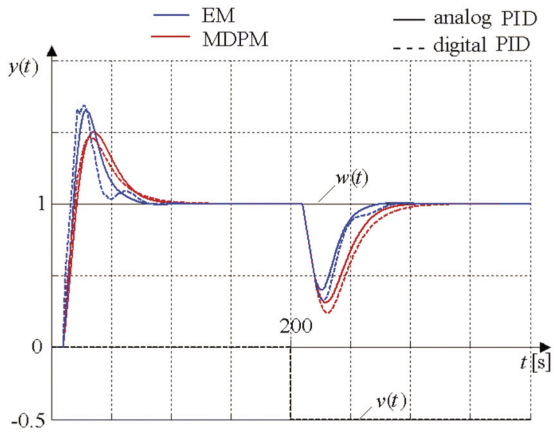

Fig. 4: *Responses of control system with analog and digital PID controllers for the multiple dominant pole method (MDPM) and experimental method (EM).*

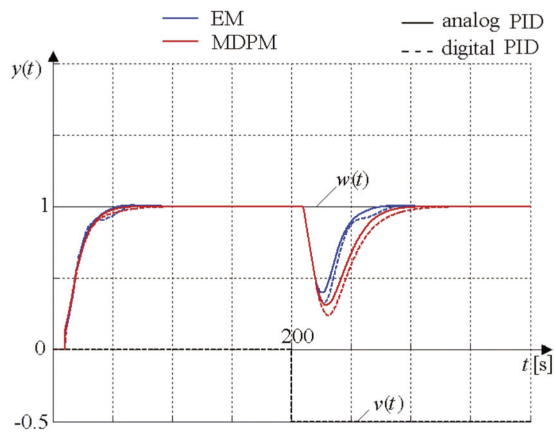

Fig. 5: *Responses of control system with analog and digital 2DOF PID controllers for the multiple dominant pole method (MDPM) and experimental method (EM).*

It is obvious that for standard controllers the servo responses have big overshoots and for 2DOF controllers the overshoots do not come up and which simultaneously cuts time down.

CONCLUSION

In the article two tuning methods for analog and digital 2DOF PI and PID controller are described.

These methods are suitable for the integrating plants with a time delay. Both methods give relatively good servo and regulatory responses. The use is shown in the example.

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