

Diagnostics of Fault in wind Turbine Rolling Element Bearing using Frequency Analysis

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Abstract: This paper is devoted to the study of wind turbine bearing failures. As maintenance costs in the wind industry are quite high, we are interested in developing approaches to minimize costs and optimize wind turbine downtimes and ensure the safety of people and property. Different techniques are used for fault analysis such as short-time Fourier transform, wavelet analysis, model-based analysis and vibration method. Each analysis technique has its preferred field of application. This paper explains the procedure for detecting bearing faults using Electromechanics analysis by characterizing each fault by a well-defined frequency. The analysis is carried out offline in MATLAB to analyse the dynamic behaviour of a ball bearing exhibiting vibration defects.

Keywords: Wind turbine, Ball bearing, Fault diagnosis, Frequency, Vibration Analysis methods, Runge–Kutta method.

1. Introduction

The bearing is an important part of a wind turbine. It is used for relative movement between the contact surfaces. The precision rolling-element bearing of the twentieth century is a product of exacting technology and sophisticated science. It is simple in form and concept, yet so effective in reducing friction and wear in a wide range of machinery [1].

As a precise mechanical component, bearings are widely used in rotor bearing systems such as aero-engines and train wheel sets (Figure 1). The quality of mechanical parts directly affects the performance of mechanical systems, and life is a comprehensive reflection of quality. The high-tech field requires each mechanical component to still have high mechanical performance, reliability and service life under harsh working conditions [2 - 5].

The smooth and quiet running of rolling bearings will directly affect the precision, performance and the reliability of the mechanical system, if the fault features information can be effectively extracted at the early stage, and accurately identifying the running state of the bearing, timely replace or repair the damaged bearings, effectively avoid cascading failures, has a great significance for reducing economic losses [6 - 8].

Condition monitoring in process control industry has got now a day's very big relevance. Diagnosing the faults before in hand can save the millions of dollars of industry and can save the time as well. It has been found that Condition monitoring of rolling element bearings has enabled cost saving of over 50% as compared with the old traditional methods. The most common method of monitoring the condition of rolling element bearing is by using vibration signal analysis. Measure the vibrations of machine recorded by velocity sensor or Accelerometer continuously which is mounted on the casing of the machine.

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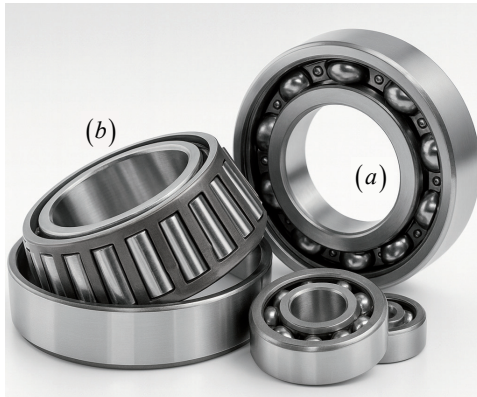


Figure1: The two types used in wind systems

(a) Ring bearing, (b) Ball bearing

More recently by taking thermal images of bearing also we can diagnose the bearing fault. But the problem in this method is that we cannot diagnose the exact location where the problem occurs. In rotating machines mainly faults occurs due to faulty bearings. IEEE analysis reveals the following fact as shown in Table 1 below [9, 10]:

Table 1: Analysis of faults

Component	Failure (%)
Bearing	40
Stator	38
Rotor	10
Other	12

Numerous studies have shown that bearing-related failures account for 40 to 60% of total failures in wind power systems [11]. From a scientific point of view, it is therefore logical to focus failure detection and diagnostic efforts on ball bearings. Moreover, several signal processing tools have been developed for acoustic analysis and vibration analysis caused by bearing failures [12, 13].

A vibration signal produced by the process, allows monitoring and making conclusions about the operational state of the machine, in addition to that allows taking appropriate measures to extend the time of use, and to minimize costs resultant from the machine's down time which results in cost effectiveness.

Bearing has Inner Race, Outer race, Balls as rolling elements. Each bearing is associated with it some characteristic frequencies which are dependent on bearing geometry. Figures (1 and 2) showed the basic elements of Bearing.



Figure 2: Example of ball bearing

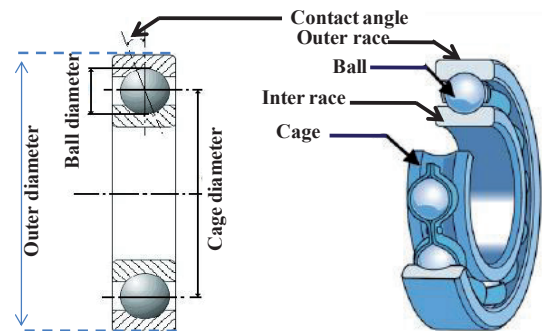


Figure 3: Sectional view of bearing

Such as ball diameter, pitch diameter, Inner race Diameter, Outer race diameter, rotational speed etc. There are number of mechanisms that can lead to bearing failure, including mechanical damages, cracks, wear and tear, lubricant deficiency and corrosion etc. Wear results in gradual deterioration of bearing components when lubrication is not proper the friction between metal to metal increases. Poor lubrication increases the bearing component temperature, which speeds up the deterioration process. Bearing that operates in an environment of high humidity may subjected to surface oxidation and produce subjected rust particles and pits. These particles can produce rapid wear.

2. Analysis Methods

The bearing defect is diagnosed from the spectrum of an acoustic or vibration measurement. In practice, there are four types of bearing defects:

- *outer ring defect,*
- *inner ring defect,*
- *ball defect;*
- *defect of the cage connecting the rolling elements.*

In vibration analysis, each type of defect has its own fundamental frequency. It is therefore possible to observe certain frequency bands and identify it. The expressions of these characteristic frequencies, when the external track is fixed, are given by [14]:

The detection of bearing defects by analysis of the electric current is relevant, but the major problem of the current analysis is that the mechanical defects are often drowned in the noise or masked by the strong electrical contribution, which does not allow a good diagnosis from the electric current. Some methods, called high frequency resolution [15, 16], have been applied to help in the detection of defects and the diagnosis of machines. The study of the effect of the bearing defect on the electrical quantities constitutes the second objective of this work. The diagnosis of the bearing defect by electrical measurement is established with the decomposition into wavelet packets and a monitoring of the variation of the energy of the coefficients likely to contain the information on the defect.

The characteristic fault frequencies can be calculated by the following equations:

» **Characteristic frequency of the inner ring [17 - 19]:**

$$f_1 = \frac{n}{2} \cdot f \cdot \left(1 + \frac{d}{D} \cdot \cos \alpha\right) \quad (1)$$

» **Characteristic frequency of the outer ring:**

$$f_2 = \frac{n}{2} \cdot f \cdot \left(1 - \frac{d}{D} \cdot \cos \alpha\right) \quad (2)$$

» **Characteristic frequency of balls:**

$$f_3 = \frac{D}{d} \cdot f \cdot \left(1 - \left(\frac{d}{D} \cdot \cos \alpha\right)^2\right) \quad (3)$$

» **Characteristic frequency of the cage:**

$$f_4 = \frac{1}{2} \cdot f \cdot \left(1 - \frac{d}{D} \cdot \cos \alpha\right) \quad (4)$$

where f is the shaft frequency, n is the number of balls, d is the ball diameter, D is bearing pitch diameter and α is the contact angle between inner race and outer race.

These characteristic frequencies allow us to extract the so-called condition indicators for the different parts of the bearing which are the inner ring, the outer ring, the cage and the balls. An observation window is usually defined around the bearing fault frequencies; in this work, the windowing used is "Hamming" and this is designed to ensure that even if the shaft speed is somewhat inaccurate, the amplitude of the bearing fault

frequency can still be captured. These indicators are in the form of energy extracted from the envelope spectrum.

Below are mentioned the four equations of the different bearing condition indicators extracted from the vibration signal [20, 21], and in the appendix, we find the Matlab code used in this work to convert the vibration signal into condition indicators capable of giving information on the bearing degradation state.

For nonlinear dynamic problems, the modeling of the bearing is equivalent to the modeling of the mass-spring assembly. In this case, the vibration analysis aims to determine the internal and external forces that are applied to an installation and to decide on the severity of their presence and their amplitudes.

The vibration analysis can be characterized by the differential equation of motion (Equation 5) of a mass - spring - damping system (Figure 4):

$$m\ddot{y} + c\dot{y} + k \cdot y = F \quad (5)$$

where m is the mass in kg, c is the damping in N.s/m, k is the stiffness in N/m, $y = y(t)$ represents the displacement of the mass in meters from its static equilibrium position, and $F = F(t)$ represents the impulse emitted by the impact of the bearing (external force) in N.

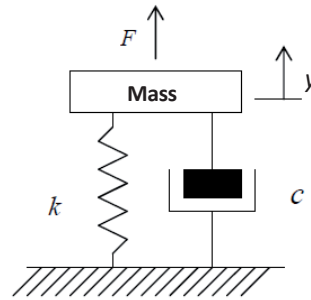


Figure 4: Equivalent bearing system

The characteristic frequency is given by the following expression:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (6)$$

where ω is the natural pulsation in rad/s, f is the frequency in Hz.

To solve the second-order differential equation (Equation 5) numerically, it is equivalent to solving the system of first-order equations:

We pose

$$y_1 = y, y_2 = y', \frac{c}{m} = 2\xi\omega_n, \frac{k}{m} = \omega_n^2, \text{ and } \frac{F}{m} = d:$$

$$\Rightarrow \begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -2\xi\omega_n y_2 - \omega_n^2 y_1 + d \end{cases} \quad (7)$$

For the resolution of the differential system (7), we used the Runge–Kutta method [22 – 26]. The algorithm is as follows:

» Given a time step h , an initial condition (x_0, y_0) and a maximum number of iterations N

» For $0 \leq n \leq N$

for $i = 1, 2, 3, \dots, l;$

$$k_{i,1} = hf_i(t_n, y_{1,n}, y_{2,n}, \dots, y_{l,n})$$

for $i = 1, 2, 3, \dots, l;$

$$k_{i,2} = hf_i\left(t_n + \frac{h}{2}, y_{1,n} + \frac{k_{1,1}}{2}, y_{2,n} + \frac{k_{2,1}}{2}, \dots, y_{l,n} + \frac{k_{l,1}}{2}\right)$$

for $i = 1, 2, 3, \dots, l;$

$$k_{i,3} = hf_i\left(t_n + \frac{h}{2}, y_{1,n} + \frac{k_{1,2}}{2}, y_{2,n} + \frac{k_{2,2}}{2}, \dots, y_{l,n} + \frac{k_{l,2}}{2}\right)$$

for $i = 1, 2, 3, \dots, l;$

$$k_{i,4} = hf_i(t_n + h, y_{1,n} + k_{1,3}, y_{2,n} + k_{2,3}, \dots, y_{l,n} + k_{l,3})$$

$$y_{i,n+1} = y_{i,n} + \frac{1}{6}(k_{i,1} + 2k_{i,2} + 2k_{i,3} + k_{i,4})$$

$$t_{n+1} = t_n + h$$

Write $\mathbf{t}_{n+1}$ et $\mathbf{y}_{i,n+1}$ for $i = 1, 2, 3, \dots, l;$

» End ■

3. Wind Turbine Modeling

A wind turbine extracts energy from moving air by slowing the wind down, and transferring this harvested energy into a spinning shaft, which usually turns an alternator or generator to produce electricity (Figure 5).

Wind is air in motion, and like anybody in motion, we can associate it with kinetic energy. If we consider that the density of air is constant, we can say that the energy provided by the wind is a function of its speed:

$$E_c = \frac{1}{2}mv^2 \quad (8)$$

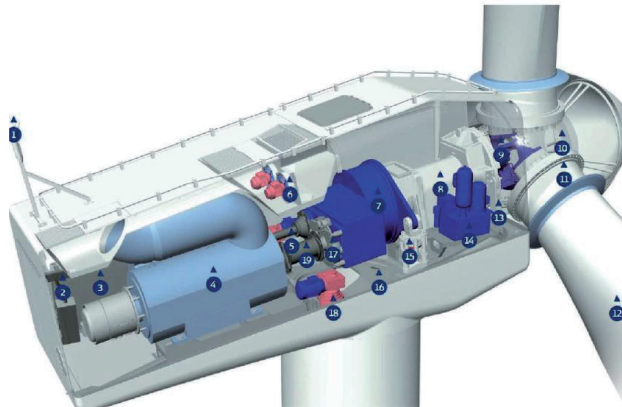
with: m mass of the air volume (in kg) and v instantaneous wind speed (in m/s).

The theoretically recoverable power for one second is given by the following expression:

$$P = \frac{E_c}{t} = \frac{1}{2}mv^2 = \frac{1}{2}\rho \cdot v \cdot S \cdot v^2 = \frac{1}{2}\rho \cdot S \cdot v^3 \quad (9)$$

with: ρ air density (kg/m^3) and S surface area of the recovery device (m^2).

Regardless of the technology used, it is impossible to recover the theoretical power in full by a wind turbine. Therefore, the recoverable power is lower, since the air must retain residual kinetic energy for a flow to remain (the Betz limit). Therefore, the maximum recoverable power is:



- 1- Ultrasonic anemometer and wind vane
- 2- Maintenance winch
- 3- Nacelle central unit
- 4- Opt speed generator
- 5- Variable pitch cylinder
- 6- Cooling system
- 7- Speed multiplier

- 8- Main shaft
- 9- Blade regulation system
- 10- Hub
- 11- Blade bearing
- 12- Blade
- 13- Rotor locking system
- 14- Hydraulic block

- 15- Torque arm
- 16- Chassis
- 17- Mechanical brake
- 18- Yield reducer
- 19- Composite coupling

Figure 5: Main elements of a wind turbine

$$P_{\max} = \frac{16}{27} \cdot P = \frac{8}{27} \rho \cdot S \cdot v^3 \quad (10)$$

The rotor converts the energy contained by the wind into mechanical energy. The following well known equation between wind speed and power extracted from the wind holds [27– 32]:

$$P_{aer} = C_p(\lambda, \beta) \cdot P = \frac{1}{2} C_p(\lambda, \beta) \cdot \rho \cdot S \cdot v^3 \quad (11)$$

The torque produced by the wind turbine is then:

$$T_w = \frac{P_{aer}}{\omega_n} \quad (12)$$

Where, ω_m is the Angular velocity of the wind turbine rotor, C_p is the power performance coefficient or aerodynamic efficiency; λ is the tip speed ration of the rotor blade tip speed to wind speed and β is the blade pitch angle (degree).

The performance coefficient C_p that is a function of the tip speed ratio λ and the pitch angle β will be investigated further. The calculation of the performance coefficient requires the use of blade element theory. As this requires knowledge of aerodynamics and the computations are rather complicated, numerical approximations have been developed. Here the following function will be used:

$$C_p = 0,5176 \left(\frac{116}{\lambda_i} - 0,4\beta - 5 \right) \cdot e^{\frac{21}{\lambda_i}} + 0,0068\lambda \quad (13)$$

$$\text{where: } \lambda_i = \left(\frac{1}{\lambda + 0,08\beta} - \frac{0,035}{\beta^3 + 1} \right)^{-1} \quad (14)$$

The tip speed ration of the rotor blade tip speed to wind speed is given by the following relation:

$$\lambda = \frac{\Omega_r \cdot R}{v} \quad (15)$$

where R is the radius of the wind turbine rotor. Ω_r is the angular speed of turbine.

The dynamic equation of the wind turbine is given by solving the following differential equation:

$$\frac{d\Omega_r}{dt} = \frac{1}{J} (T_g - T_{em}) \quad (16)$$

where Ω_r is the angular speed of the generator, J is the moment of inertia of the generator and the turbine, T_g is the torque developed by the wind

turbine brought back to the generator side, and T_{em} is the generator electromagnetic torque.

4. Results and Discussion

The device studied in this work is composed of a wind turbine with a nominal power of 1.5 MW, comprising blades with a radius of 37.5 m which drives a generator through a multiplier.

The various parameters used in this work are listed in table (2). These values are approximate and may vary markedly as a function of wind turbine proprieties and bearing characteristics.

All bearings are made of highly resistant materials and should have a high modulus of elasticity to withstand all stresses and for a long service life. To model our bearings, it is necessary to know the properties of the materials constituting the ball bearings. The bearing parameters used in this investigation are listed in table (3).

Table 2: Characteristics of wind turbine used in simulation calculations

Characteristic	Value
Rotor diameter	75 m
Area covered by rotor	4418 m ²
Blade	3
Rotor speed	9 – 21 rpm
Nominal power	1.5 MW
Nominal wind speed	12 m/s
Cut-in wind speed	4 m/s
Stop wind speed	25 m/s
Gear box ratio	80
Regulation	Pitch/optiSpeed
Total of moment inertia	5,9.10 ⁶ kg.m ²
Voltage and frequency	690 V/50 Hz
Air density	1.23 kg/m ³

To determine the wind variations on the El Bez site of the city of Setif in Algeria, we followed the evolution of the wind speed during each day of the month of October 2024. Each point on the X axis of figure (6), is the average value of a day.

This first step of this work, allows optimizing the design of wind turbines to minimize the costs related to the production of electricity.

Table 3: Parameters of bearing used in simulation calculations

VEX70 7 CE1 UL Angular contact ball bearings, super-precision dimensions	
Characteristic	Value
Outside diameter	110 mm
Inside diameter	70 mm
Thickness	20 mm
Contact Angle	15°
Number of balls	12
Ball diameter	9.52 mm
Mass of bearing	0.13 kg
Young's modulus of ball	315 GPa
Young's modulus of rings	210 GPa
Poisson's ratio of balls	0.26
Poisson's ratio of rings	0.3
Density of balls	3190 kg/m ³
Thermal expansion coefficient of steel	11,7.10 ⁻⁶ at 20°C
Angular velocity of inner ring	2*pi*NT/60 (tr/mn)
Angular velocity of outer ring	0 (tr/mn)
Lubricant viscosity	1,13.10 ⁻⁸ N.s/m ²
Modulus of elasticity of rings	219000 N/m ²
Modulus of elasticity of ball	202.10 ³ N/m ²
Poisson's ratio of rings	0.3
Poisson's ratio of ball vb	0.3
Basic dynamic load	10.6 kN
Basic static load	5 kN
Reference speed	26 000 tr/min
Limiting speed	24 000 tr/min

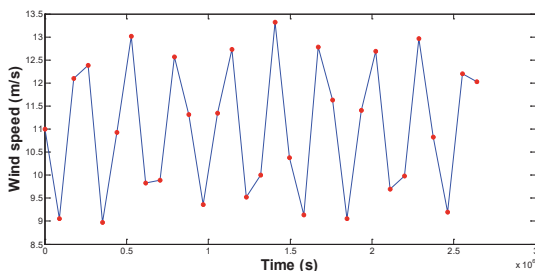


Figure 6: Wind means velocity at Setif department for October 2024

The performance coefficient or power coefficient indicates the efficiency with which the wind turbine converts the mechanical energy of the wind into electricity. This coefficient differs depending on the

turbine. We start by taking the power curve and dividing it by the swept area of the rotor to obtain the output power per square meter of swept area. We then divide this result by the wind power per square meter for each wind speed.

The graph of figure (7) represents the power coefficient curve for a wind turbine. Although the average efficiency of such a wind turbine is normally greater, it varies considerably depending on the wind speed.

The power coefficient takes into account the Betz limit, which is why it will always be lower than this limit; it is of the order of 35% for a horizontal axis wind turbine and it does not exceed 10% for a vertical axis wind turbine.

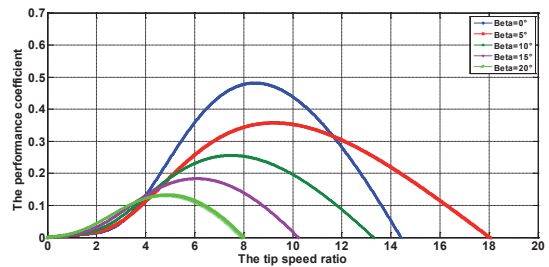


Figure 7: Performance coefficient C_p , as a function of tip speed ratio λ and pitch angle β

A wind packet of mass m flowing at an upstream speed v in the axial direction of the wind turbine has kinetic energy. This represents the total power available for extraction (Figures 8 - 9).

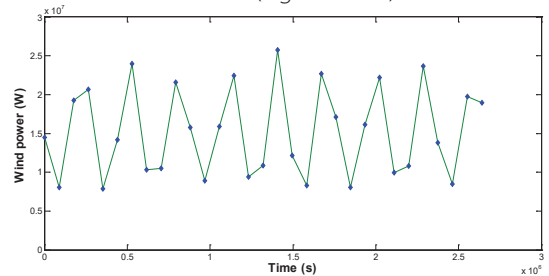


Figure 8: Power variation during the month of October 2024

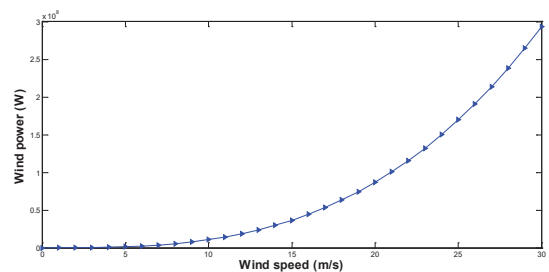


Figure 9: Total power of the wind

The variations of the extracted power are shown in figure (10). The fraction of power extracted from the available power in the wind, by practical turbines is expressed by the coefficient of performance C_p .

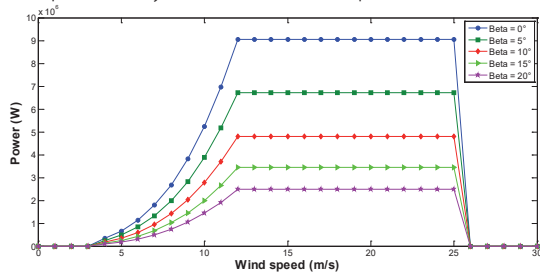


Figure 10: Wind power

Note that, for this work, the wind turbine starts at around 3 - 4 m/s, then the wind speed increases, the power increases and arrives at a certain speed which will be called the nominal speed. It will be necessary to brake the wind turbine because the tip of the blade turns at a dangerous excessive speed. We will be obliged, beyond this speed, to brake the wind turbine (limit the power of the wind turbine). Beyond a certain speed which is of the order of 25 m/s approximately, we stop the wind turbines because we can no longer brake them.

The calculation of the torque in wind turbines is very important for the dimensioning of the system energetically. Because, there is no power without an angular speed and a torque. So, it is obvious that it is interesting to be interested in the torques that we will be able to recover on the motor shaft of the wind turbine. The variations of the torque of a wind turbine in the absence and presence of bearing defects are shown in figures (11, 12).

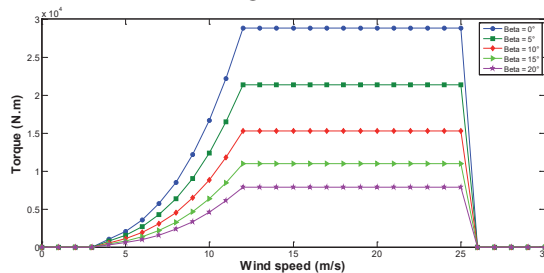


Figure 11: Wind turbine torque without bearing fault

From an energy point of view, damping corresponds to a progressive dissipation of the initial mechanical energy of the system.

If the mass is pulled downwards, the spring tends to bring the mass back to its static equilibrium

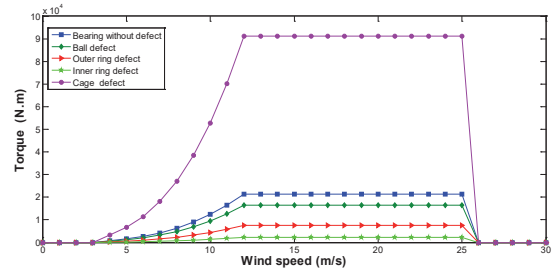


Figure12: Wind turbine torque in the presence of different defects in the ball bearing

position. Once it reaches this position, it will have acquired maximum kinetic energy which pushes it to continue its movement until its speed is cancelled. The oscillator considered here is of a mechanical nature with one degree of freedom.

The numerical resolution of the second order differential equation allows us to find the results presented in figures (13 – 17) by considering the initial conditions to be zero.

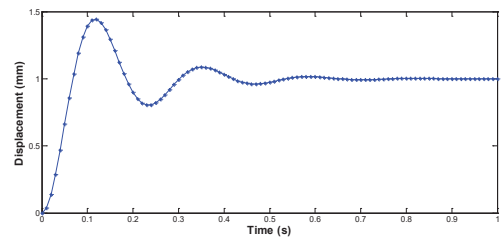


Figure 13: Displacement without bearing fault

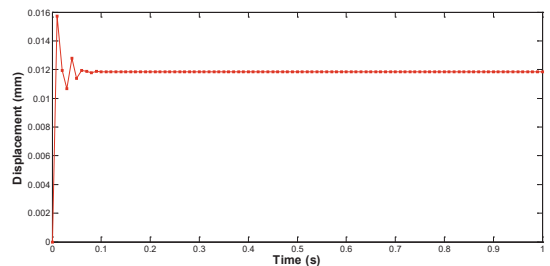


Figure 14: Displacement in the presence of inner ring defect

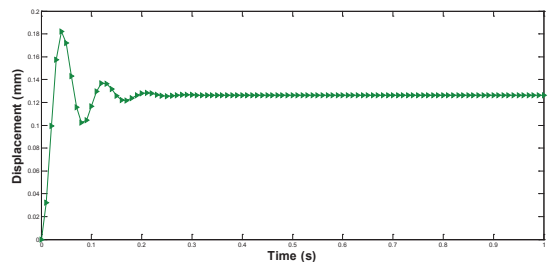


Figure 15: Displacement in the presence of outer ring defect

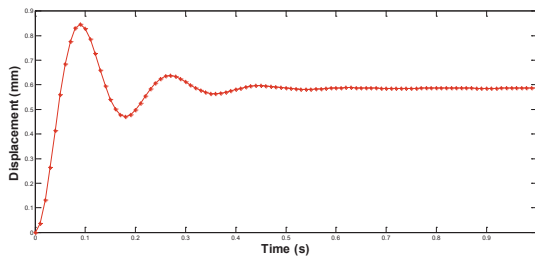


Figure 16: Displacement in the presence of balls defect

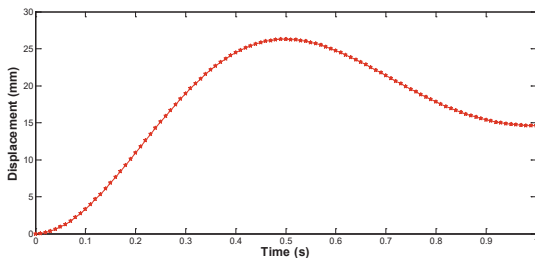


Figure 17: Displacement in the presence of cage defect.

Analysing the results found, we can conclude:

- *The cage defect generates a new torque on the shaft that is 400% higher than the values without defect with a huge displacement. It is recommended to stop the process so as not to completely destroy the installation.*
- *Inner or outer defects lead to very small values of torque. So, the efficiency will be very low and the economic cost increases sharply with low displacement. It is recommended to change the defective part.*
- *The defect in the balls reduces the efficiency a little. It is recommended to scheduled maintenance as soon as possible.*

The ball bearing is the most used in the industrial world because it has the best performance-price ratio. We can note two types of damage characteristic of the deterioration of bearings: so-called natural damage due to bearing fatigue and damage due to poor assembly or use of the bearing. The most common case is a spalling defect due to bearing fatigue. Fatigue spalling is a normal phenomenon that leads to failure, regardless of the conditions of use and operation.

The rotating elements of a wind turbine, during operation, generate internal forces and deformations within the structure that vary according to the rotation frequency and ball bearing defects. These deformations and forces cause the structure to move relative to itself, which constitutes vibration.

4. Conclusion

The operation of many wind turbine applications depends on the bearings they contain. If one

bearing fails, the entire system shuts down and downtime quickly adds up. Bearing fault diagnosis is important in condition monitoring of any rotating machine. Early fault detection in machineries can save millions of dollars in emergency maintenance cost.

The presence of defects in turbines results in severe vibration of rotating machinery. Timely detection of these defects and estimation of the remaining service life are the concerns of researchers because sudden failures of bearings can cause malfunction of the entire system, or shutdown, resulting in economic loss.

In order to quantify the severity of bearing failures and defects, the monitoring technique used must not be limited to the detection of defects, but also to the performance of a prognosis and an in-depth diagnosis that allows the location of failures and breakages and the calculation of the remaining life of the bearing. Vibration analysis is a technique that allows the performance of this prognosis and diagnosis.

We find that the dynamics of the elements of a ball bearing become increasingly important at high speeds.

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