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Mathematical Model of Fatigue for Steel Wire Ropes

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BIOGRAPHICAL NOTES

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KEY WORDS

Steel Wire Rope, Rope Fatigue, Mathematical Model

ABSTRACT

The steel wire rope is stressed during operation by the tensile and bending forces. This loading state has a relevant impact on mechanical characteristics of the rope. A negative influence on the rope properties has a corrosion of the rope wires, especially. The wires are the basic construction components of the rope. It is necessary to know changes of rope mechanical characteristics with regard to a safety and reliability of the rope operation. In a case that the rope properties are not able to fulfil requirements of the safety and reliability, it is unavoidable to put-off such rope. There is presented in this paper a mathematical model describing dependence between prolongation of the rope and the number of loading cycles at various conditions. Such model enables to predict properties of the tested steel wire rope, as well as to estimate the put-off time of the rope.

1. Introduction

The steel wire rope is stressed and subjected to the corrosion during the current operation. There are changing mechanical properties of the rope due to the above-mentioned factors. This fact has a negative influence on the operational safety and reliability. Therefore it is important to identify process of changing of the rope characteristics and, above all, to define correctly the time point of a dangerous or unreliable operation, i.e. to determine the rope put-off time. There was developed a mathematical model of the rope prolongation in relation to the number of working cycles using the experimental results obtained from three samples of the rope. This model enables to estimate mechanical properties of the tested rope, as well as the rope putoff time, eventually.

2. Parameters of the Tested Ropes

The three rope samples, manufactured in Drôtovní, a.s. Hlohovec [8], were tested on the fatigue-testing machine. Each of the samples is wound unidirectionally with the next parameters [7]:

Table 1: Rope parameters.

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Rope cross-sectional construction	6 (1+9+9) + v
Nominal tensile strength of wires	1570 MPa
Nominal rope loading capacity	156,5 kN

Table 2: Wire diameters.

Nominal wire diameters	
Core of strand	1,6 mm
The 1st layer	0,71 mm
The 2nd layer	1,25 mm

The individual rope samples were tested at various conditions. The first sample was a new rope without primarily and consequently lubrication, with the length necessary for test on the fatigue-testing machine. Another two samples were situated into a corrosive ambient created by the 5% solution of the NaCl during 6 months [3]. After the

Table 3: New rope without primary and subsequent lubrication.

k	a_0	a_1	a_2	a_3	a_4	a_5	a_6	$\sum (d_i - d(n_i))^2$
1	202,98	0,852						349,4
2	199,66	1,909	-0,0533					288,9
3	193,58	6,158	-0,6066	0,0187				95,0
4	190,46	10,268	-1,6070	0,0995	-0,0021			36,4
5	189,04	13,599	-2,9218	0,2853	-0,0129	0,00022		19,9
6	188,55	15,567	-4,0780	0,5354	-0,0375	0,00134	-0,000019	16,9

Table 4: Rope lubricated primarily without subsequent lubrication after 6 months in corrosive ambient.

k	a_0	a_1	a_2	a_3	a_4	a_5	a_6	$\sum (d_i - d(n_i))^2$
1	86,13	1,849						1061,2
2	81,25	3,313	-0,0697					923,4
3	68,76	11,408	-1,0562	0,0313				74,9
4	68,96	11,171	-1,0025	0,0273	0,0001			74,6
5	66,84	15,608	-2,6183	0,2390	-0,0114	0,00022		39,1
6	67,23	14,209	-1,8625	0,0879	0,0024	-0,00036	0,000009	37,3

first three months one sample was lubricated subsequently. After 6 months both rope samples were tested at the fatigue-testing machine. The fatigue test was performed with one rope lubricated primarily, however without subsequently lubrication and with the second rope, which was lubricated primarily, as well as subsequently during operation. The producer of ropes applied the lubricant Elaskon and the same kind of lubricant was used for subsequently lubrication [7]. During the testing process was measured the prolongation d of the rope depending on the number of loading cycles n . Note: in this paper there are given the values n in thousands of loading cycles and the values d are in millimetres.

3. Mathematical model

There was chosen a regressive function for each of the samples for simulation of the relation between the rope prolongation d and the number of loading cycles n according to the preliminary evaluation of the measured values. The regressive function is a polynomial

$$d(n) = a_0 + a_1 n + a_2 n^2 + \dots + a_k n^k, \quad (1)$$

$k \geq 1$, whereas it was defined for each of the samples the regressive function (1) for $k=1;2;3;4;5;6$. The coefficients of the corresponding regressive function was found by means of the least square

method. In the tables 1, 2 and 3 are presented the obtained results, including summations of the residual squares, where d_i is the value of prolongation measured for the number of loading cycles n_i .

From the tables 1, 2 and 3 it is evident that in the case of all samples are summations of the residual squares for $k = 1, 2, 3$, and 4 very large numerical values and they are significantly higher predominately than for $k = 5$ and 6, so the regressive models are not suitable for $k = 1, 2, 3$, and 4. Summations of the residual squares for $k = 5$ and 6 are low numerical values sufficiently and they are different

only few each other. Therefore it is possible, for simulation of the relation between the rope prolongation d and the number of loading cycles n , to chose a regressive function for each of the samples for $k = 6$, i.e.

$$d(n) = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + \dots + a_6 n^6. \quad (2)$$

The satisfactory accuracy is verified also by means of the graphs of the measured values and dependences (2) presented on the Figs. 1, 2 and 3.

Table 5: Rope lubricated primarily and subsequently after 6 months in corrosive ambient.

k	a_0	a_1	a_2	a_3	a_4	a_5	a_6	$\sum (d_i - d(n_i))^2$
1	52,99	1,217						1570,9
2	50,38	1,904	-0,0287					1529,4
3	35,27	10,623	-0,9833	0,0272				254,9
4	34,11	11,861	-1,2363	0,0445	-0,0004			246,6
5	29,23	20,807	-4,1744	0,3935	-0,0175	0,00030		62,3
6	29,75	19,224	-3,4060	0,2546	-0,0061	-0,00014	0,000006	59,4

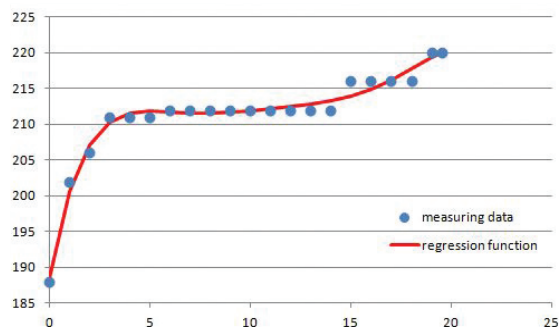


Fig. 1: New rope without primary and subsequent lubrication.

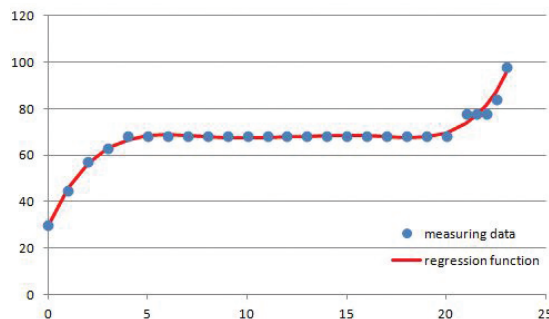


Fig. 3: Rope lubricated primarily and subsequently after 6 months in corrosive ambient.

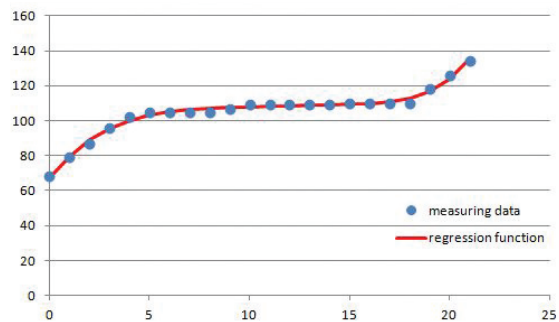


Fig. 2: Rope lubricated primarily without subsequent lubrication after 6 months in corrosive ambient.

4. Conclusion

It is possible to claim that the created regressive model is suitable and it enables to describe the dependence between the rope prolongation and the number of loading cycles with a sufficient accuracy. It can be also stated that for each of the samples exists an interval $\langle c_L; c_U \rangle$, with a constant character of the function $d(n)$ or with a linear character with only a very small linear coefficient. This is validated also by means of a fact that for each of the samples exists an interval with the values of the first derivation $d'(n)$ and the second derivation $d''(n)$ of function $d(n)$, which are very closed to the zero value (see illustrative Figs. 4 and 5).

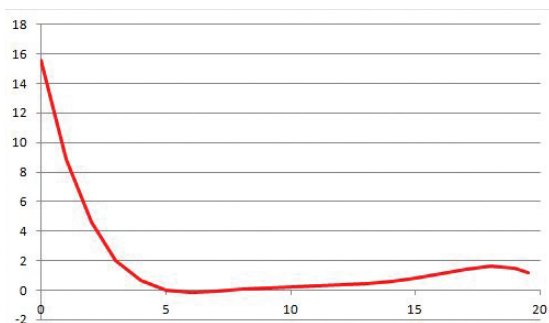


Fig. 4: The first derivation of function $d(n)$ for new rope without primary and subsequent lubrication.

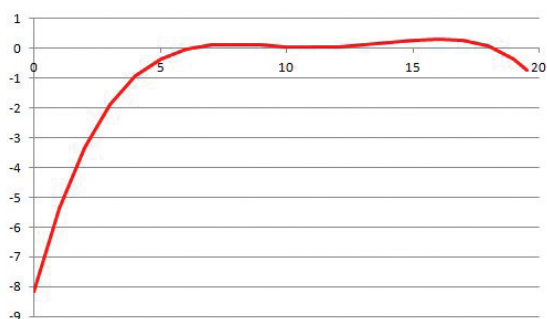


Fig. 5: The second derivation of function $d(n)$ for new rope without primary and subsequent lubrication.

There is defined in the work [5] the time point for the rope put-off like a moment of rope prolongation over 10% in comparison to the "stable" value. Determination of the rope put-off time can be defined alternatively like a such time point, which the function $d(n)$ begins to lose its constant or linear character in. It can be applied also a fact that in a certain moment the values of the first derivation $d'(n)$ and the second derivation $d''(n)$ of the function $d(n)$ are close to the zero value. How it was mentioned already, the established regressive model is a proper model and it makes possible to describe the mutual relation between the rope prolongation and the number of loading cycles with a conforming accuracy. So, it can be applied for an estimation of the loaded rope parameters, including prediction of the time-point for the rope put-off. However, it should be performed the more detailed statistical analysis based on a larger number of measurements of the same rope samples in order to verify the proximity level of a conformity between the model and the practice.

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